Physics 212 Lenses and Mirrors Review Notes

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1 Real and Virtual

When your eye receives light rays, it attempts to extrapolate back to the point where the rays converge, and to interpret this convergence point as the location of the object which produced the light rays. If this extrapolated location is a point through which the light rays have *actually passed* before reaching the eye then these light rays have come from a real object, or a so-called "real image" of an object generated by some intervening optical elements. If the extrapolated location is a point through which the light rays have never passed (or have not yet reached upon hitting the eye), then the object or image from which the rays have come is considered to be virtual. A similar distinction between real and virtual applies when a lens or mirror is receiving the light rays instead of your eye.

The distance between a real object or image and the optical element is considered to be positive. The distance between a virtual object or image and the optical element is considered to be negative.

Finally a lens or mirror which, upon receiving parallel incoming light rays, creates a real image, is considered to have a postive focal length. A lens or mirror which creates a virtual image from parallel incoming light rays is considered to have a negative focal length.

2 Characteristics of Optical Elements

All of the lenses and mirrors we deal with in this course are "spherical", which means we've cut a chunk out of a thin hollow sphere and made it into an optical element. The radius of the sphere is referred to as the radius of curvature, R, and the focal length of a spherical element is:

$$|f| = \frac{R}{2} \tag{1}$$

where the sign of the focal length is determined as described above.

A "concave" mirror or lens is curved inward with respect to the incoming light (like a cave) and a convex mirror or lens is bowed outward with respect to incoming light. Concave mirrors and convex lenses have $f > 0$ – they are "converging" since incoming parallel rays are brought together at the focal point. Convex mirrors and concave lenses have $f < 0$ and are referred to as "diverging", since incoming parallel rays are sent off in all different directions. (Their apparent convergence point will be the focus on the "wrong side" of the mirror.)

3 Equations

If we have light rays from an object (or from the image, real or virtual, formed by another optical element) then the position of the image of this object s' formed by an element of focal length f will be related to the position of the object s by:

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \tag{2}$$

The height h' of the image will be related to the height h of the object by:

$$m = -\frac{s'}{s} = \frac{h'}{h} \tag{3}$$

where m is the magnification. If the sign of the magnification is negative then the image will be flipped upside down with respect to the original object. A real image of a real object will be upside down, and a virtual image of a real object will be right-side up.

The power of the optical element is simply a measure of its capacity to bend light rays. A higher-power optical element bends light rays more. The definition of the power for optical elements is:

$$P = \frac{1}{f} \tag{4}$$

The units of lens power are m^{-1} which in optics terminology are called diopters.

Note that a flat mirror (or lens) has an infinite radius of curvature and thus an infinite focal length. So the power of this optical element is zero, the object and image locations are related by $s = -s'$ (a virtual image) and the magnification is $m = 1$ (an upright image of identical size to the object).

4 Constructing a Lens

If we make a lens out of a chunk of glass with index of refraction n_{g} and put it in a medium with index of refraction n_{m}, then the focal length of the lens in that medium is:

$$\frac{1}{f} = \left(\frac{n_{\mathrm{g}} - n_{\mathrm{m}}}{n_{\mathrm{m}}}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{5}$$

where R_1 and R_2 are the radii of curvature of the two sides of the lens. These radii of curvature are signed numbers, and the sign of each is determined by whether the lens face is concave or convex with respect to the incoming light. (Note that if you send light in from the opposite side, we will still find the same focal length, because the *ordering* of the two radius of curvature terms will be reversed, and the *sign* of each radius of curvature will also be flipped.)

As an example, take an ordinary magnifying glass lens (both sides are bowed outwards so that the middle of the lens is fatter than its edges) with $|R_1| = 5$ cm and $|R_2| = 3$ cm made of glass with index of refraction n_{g} and placed in air ($n_{\mathrm{m}} = 1$). If light is incident on the side of radius R_1, then that side is convex with respect to the light and $R_1 > 0$. The other side will then be concave with respect to the light and so $R_2 < 0$. So for this lens:

$$f = \left(\left(\frac{1.5 - 1}{1}\right)\left(\frac{1}{5\ \mathrm{cm}} - \frac{1}{-3\ \mathrm{cm}}\right)\right)^{-1} = 3.75\ \mathrm{cm} \tag{6}$$

Finally, note that for a lens placed in air or vacuum $n_{\mathrm{m}} = 1$ and so:

$$\frac{1}{f} = (n_{\mathrm{g}} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{7}$$

Placing this lens in any medium with a larger index of refraction will decrease the numerator and increase the denominator of the index of refraction factor and so will increase the focal length of the lens (decreasing its power).