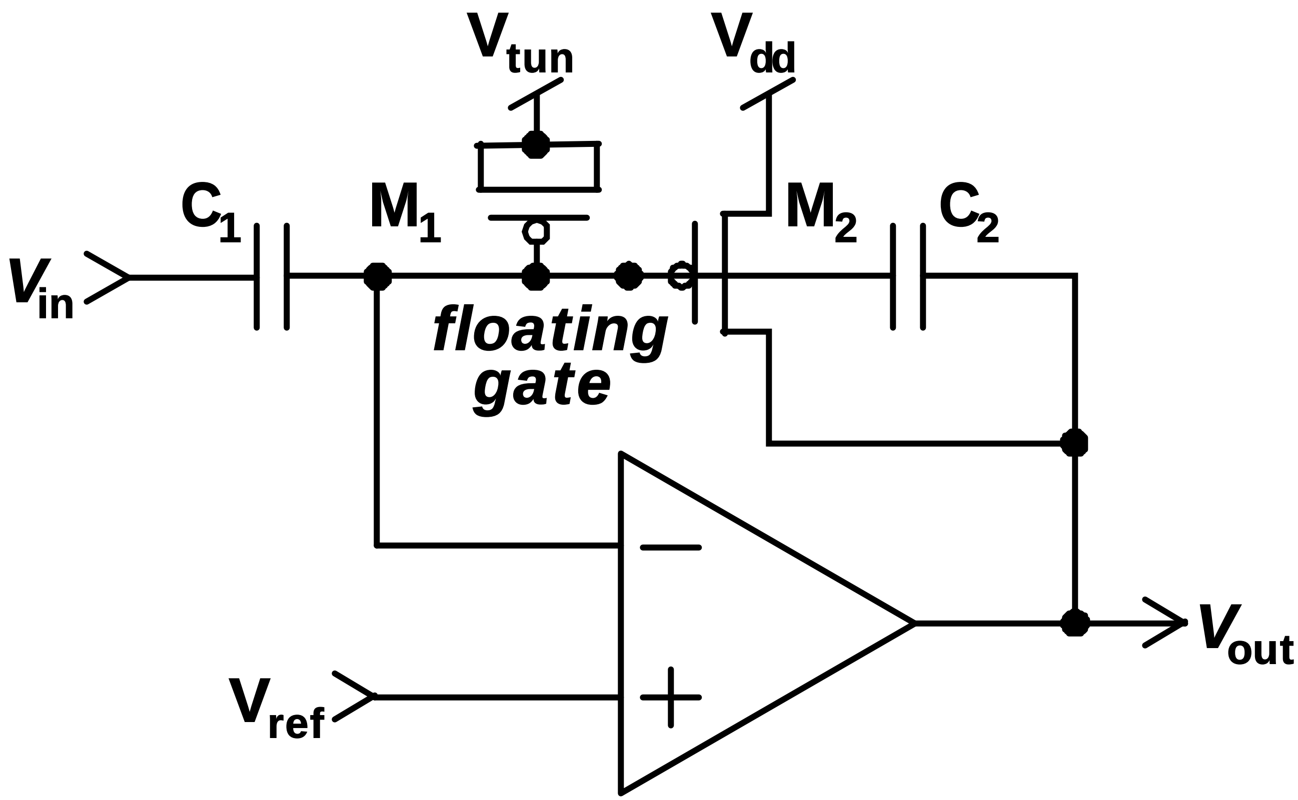
Some applications with FGs

- Autonulling amplifier (Hasler et al 2001, Rahimi et al 2002)
- Learning with probabilities (Hsu)
- Mismatch reduction (Mitros)
- Automaximizing bump circuit, and learning the means of a mixture of Gaussians (Hsu)

Autonulling Amplifier



- AC coupled with a ~0.1Hz corner
  - Tunneling/IHEI sets DC output voltage
  - Capacitive ratio -C1/C2 sets amplifier gain

see Rahimi et al, 2002

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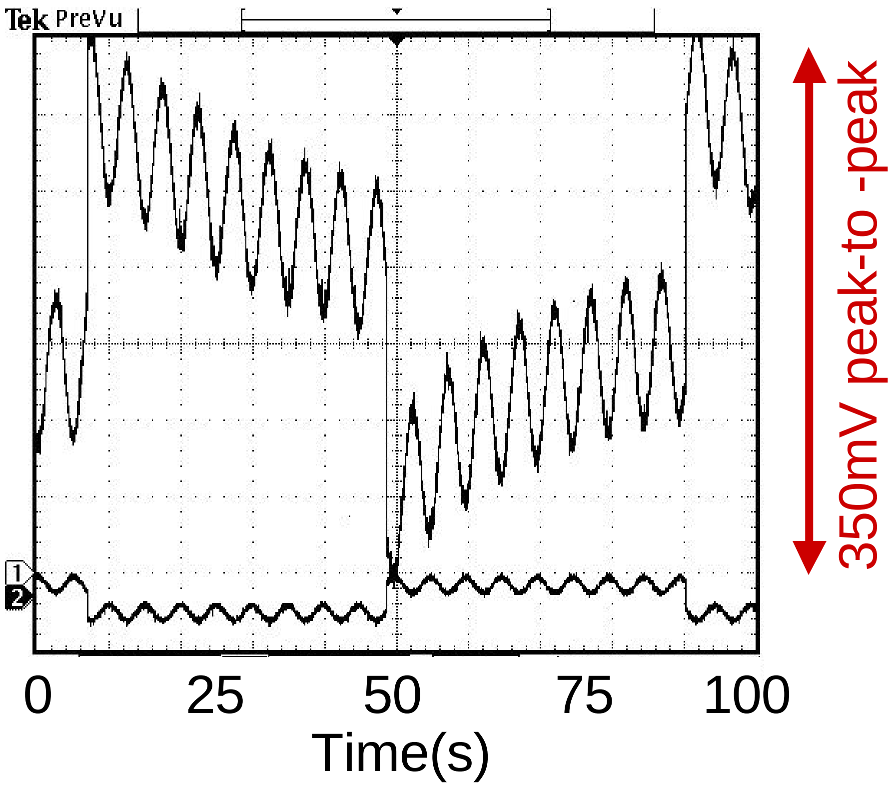
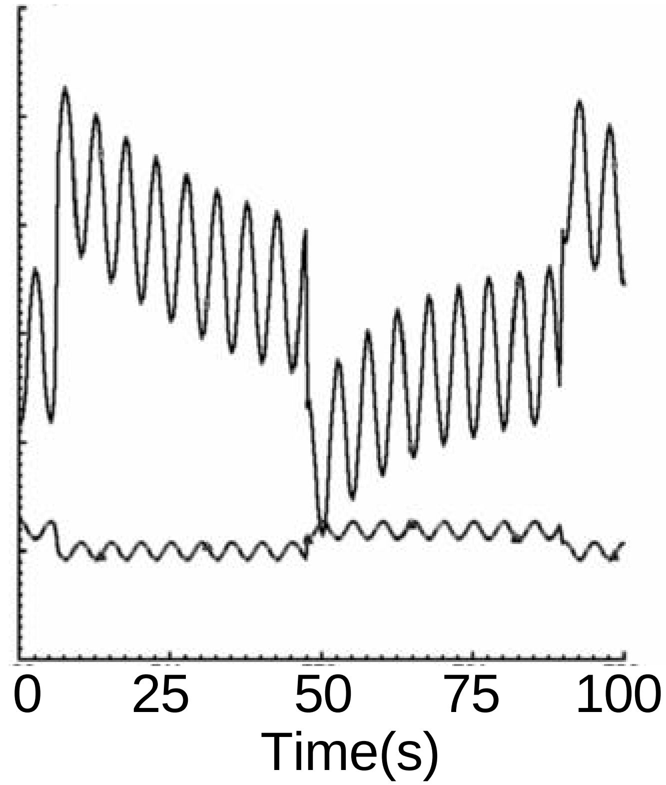
Amp Circuit Data and Simulation

Measured circuit response



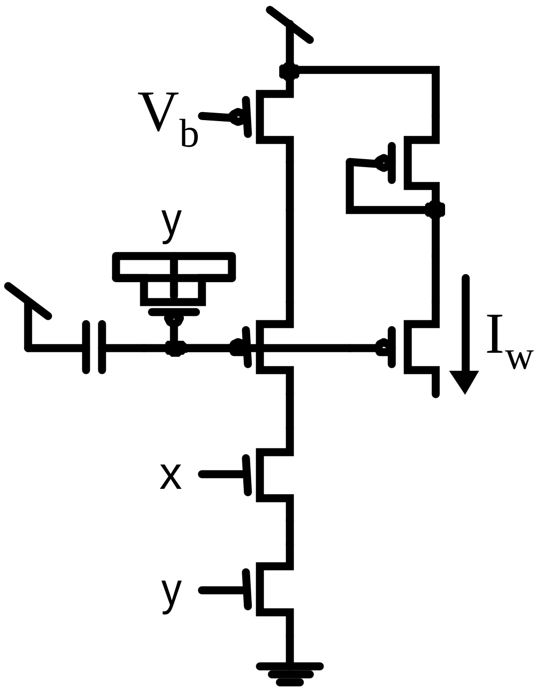
SPICE simulation



- Input: 0.2 Hz, 15mV sinewave superimposed on a 0.012 Hz, 19mV square-wave

see Rahimi et al, 2002

Learning a Conditional Probability



$$\boxed{\begin{aligned} P(V_{tun}) &= P(y) \\ P(V_{inj}) &= P(x,y) \end{aligned}}$$

$$I_{out} = I_o e^{-V_{FG}}$$

$$I_{inj} = l\,(e^{-V_{FG}})^f P(X,Y) = l\, I_o^f I_{out}^{-f} P(X,Y)$$

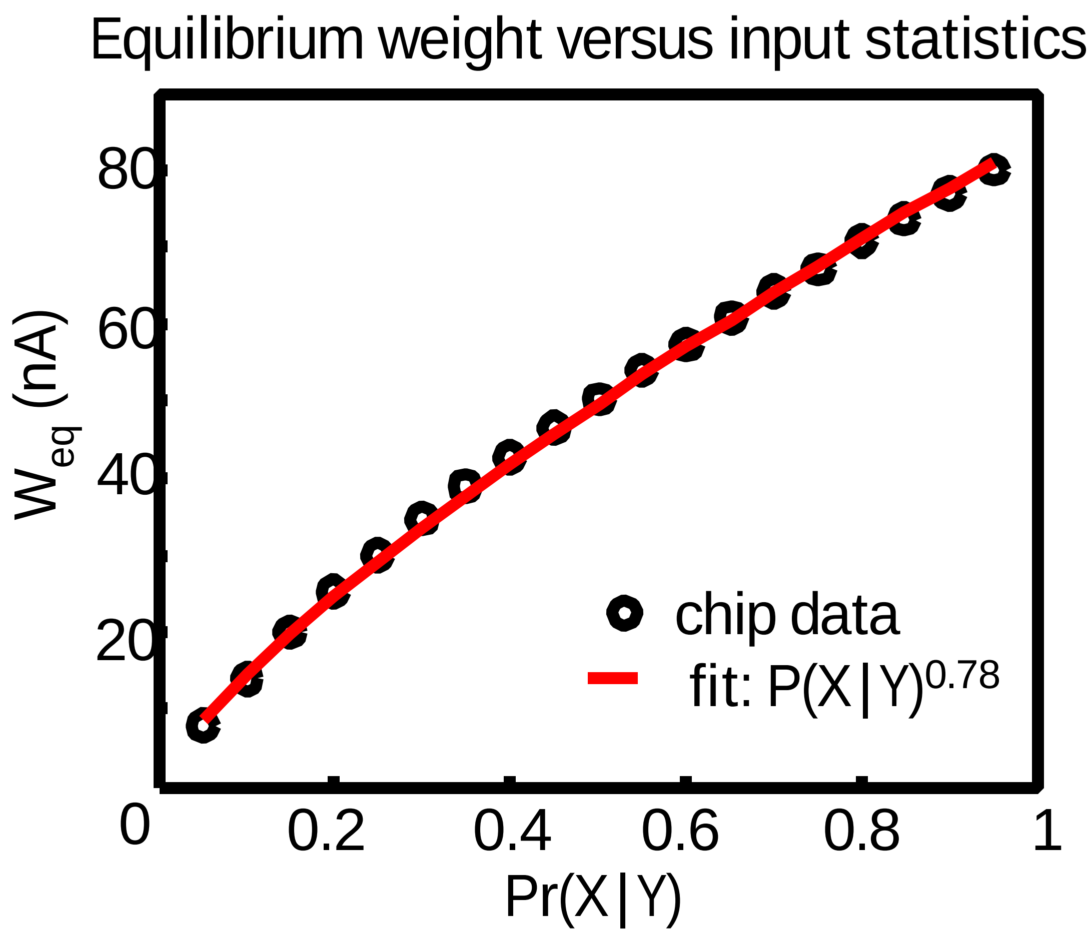
$$I_{tun} = h\,(e^{V_{FG}})^n P(Y) = h\, I_o^{-n} I_{out}^n P(Y)$$

$$I_{inj} = I_{tun} \rightarrow I_{out} = I_o \left(\frac{l}{h} \frac{P(X,Y)}{P(Y)} \right)^{\frac{1}{f+n}}$$

$$\boxed{I_w \propto \left(\frac{P(V_{inj})}{P(V_{tun})} \right)^s \propto \left(\frac{P(x,y)}{P(y)} \right)^s \propto (P(x\,|\,y))^s}$$

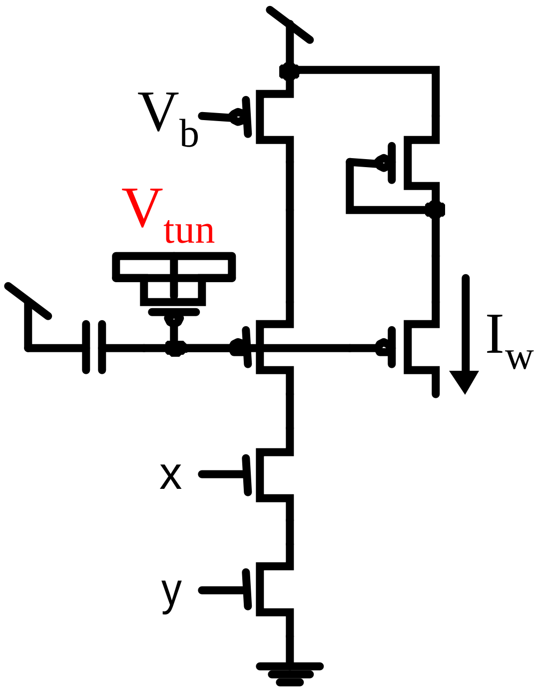
"Learning Spike-Based Correlations and Conditional Probabilities in Silicon"
Aaron Shon, Dave Hsu, and Chris Diorio. *In Proc. NIPS 2001*

Circuit data for P(X|Y)



"Learning Spike-Based Correlations and Conditional Probabilities in Silicon"
Aaron Shon, Dave Hsu, and Chris Diorio. *In Proc. NIPS 2001*

Learning Correlations



Let V_{tun} be constant (like leakage current). The circuit learns the correlation between X and Y.

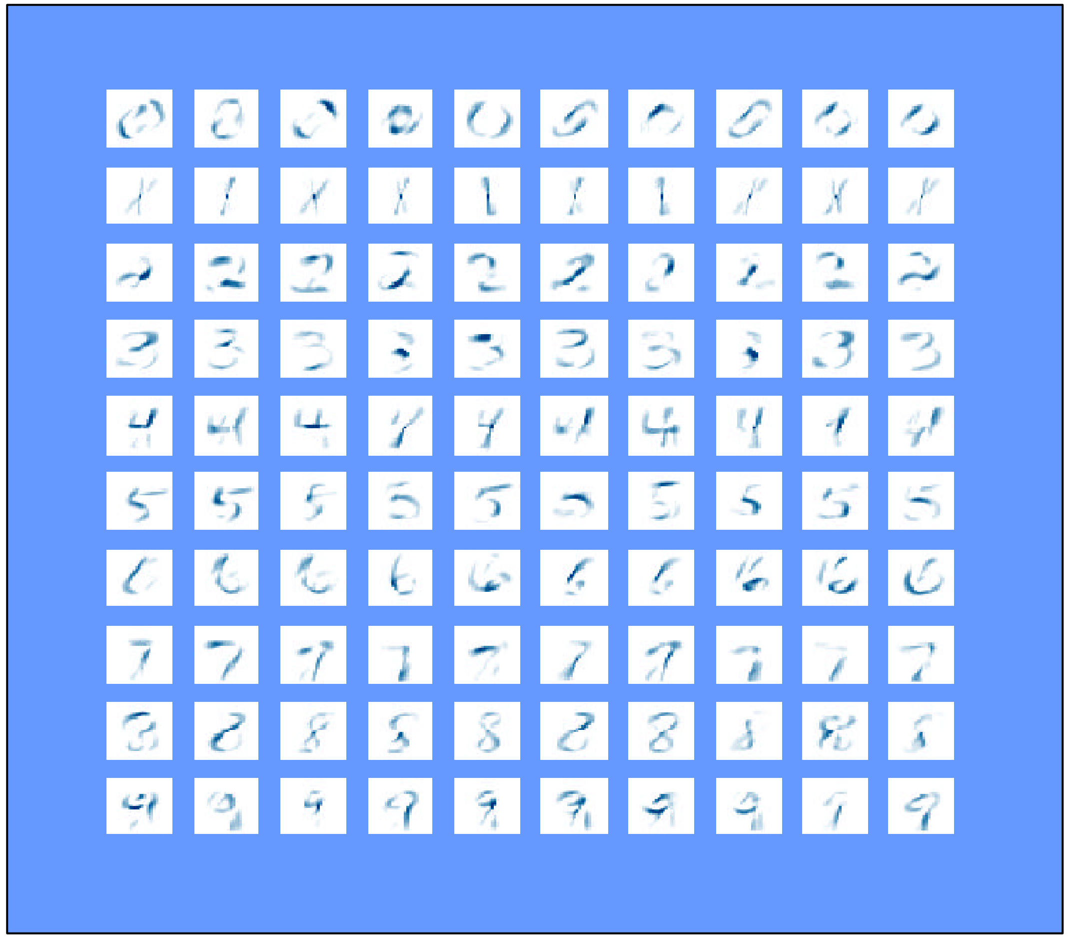
$$I_w = \left(P\left(V_{inj}\right)\right)^s = \left(P\left(x, y\right)\right)^s$$

"Learning Spike-Based Correlations and Conditional Probabilities in Silicon"
Aaron Shon, Dave Hsu, and Chris Diorio. *In Proc. NIPS 2001*

Learning on handwritten digits

- Simulated Boltzmann machines on handwritten digits
- Hidden layer neurons learn informative features
- Classification results
  - Silicon learning rule: 94.09%
  - Standard Boltzmann rule 93.88%

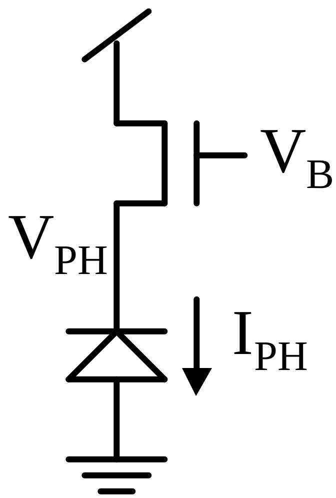
Hidden layer weights

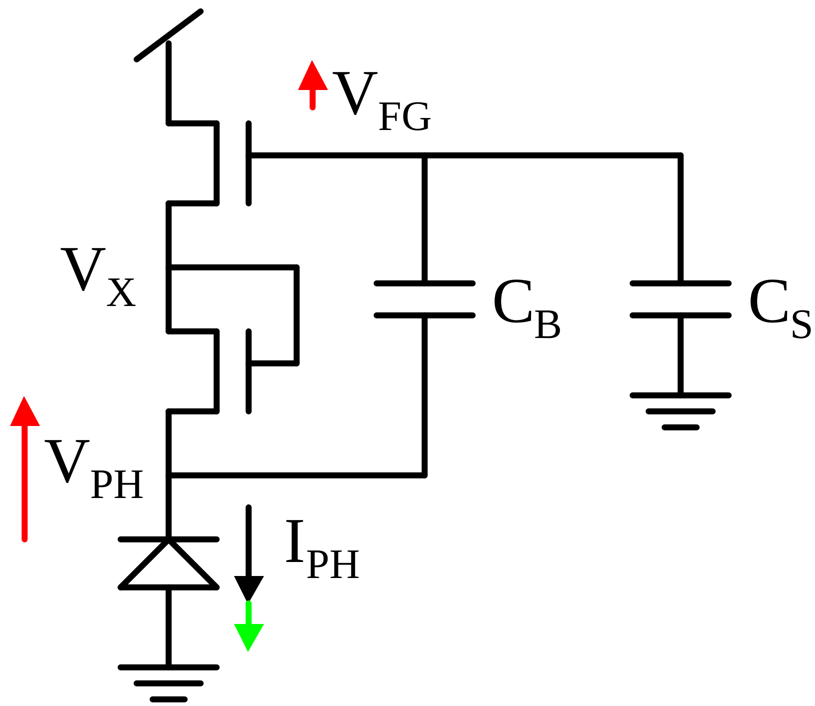


Mismatch reduction

- Standard techniques for improving mismatch require a lot of die area:
  - dummy transistors
  - common-centroid layout
  - large transistors (or multiple copies)
- Charge on floating gate is equivalent to changing V_{TH}. Floating gates allow use of minimum size transistors and programming away of most of the mismatch as long as mismatch is mainly due to V_{TH} mismatch rather than gain mismatch (true for circuits I've dealt with).

Logarithmic photoreceptors

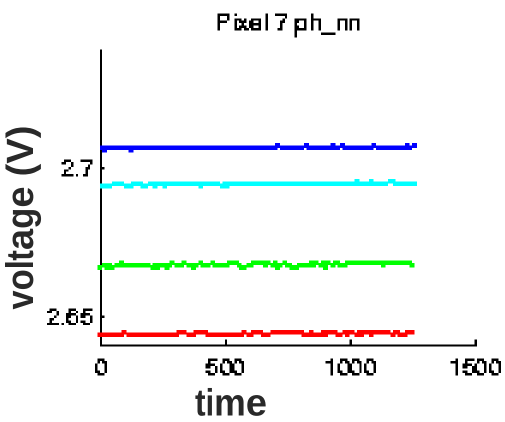




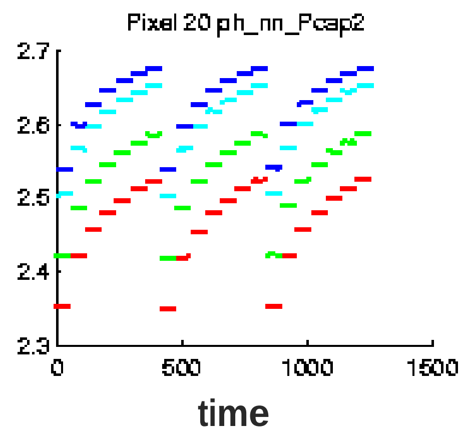
$$V_{PH} = -\log\left(\frac{I_{PH}}{I_O}\right)$$

$$V_{PH} = -2\left(\frac{C_S + C_B}{C_S}\right)\log\left(\frac{I_{PH}}{I_O}\right)$$

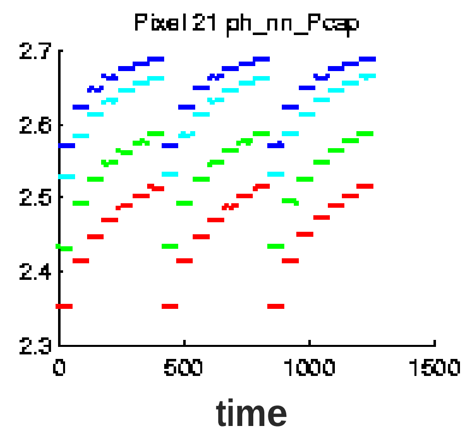
Photoreceptor: shifting DC level



No FG



with FG



with FG

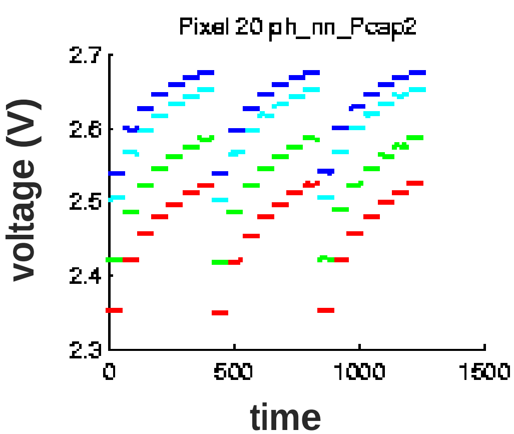
The data collection procedure was as follows: 6 tunneling pulses, 1 injection pulse, 6 tun, 1 inj, 6 tun. Several data points were collected between each programming pulse. The plots show data as a function of time. Each upward vertical jump in the data corresponds to the result of a tunneling pulse. Each downward jump corresponds to injection. Colors represent intensity of LCD monitor: Dark blue=0 (darkest), Cyan=21, Green=42, Red=63 (brightest).

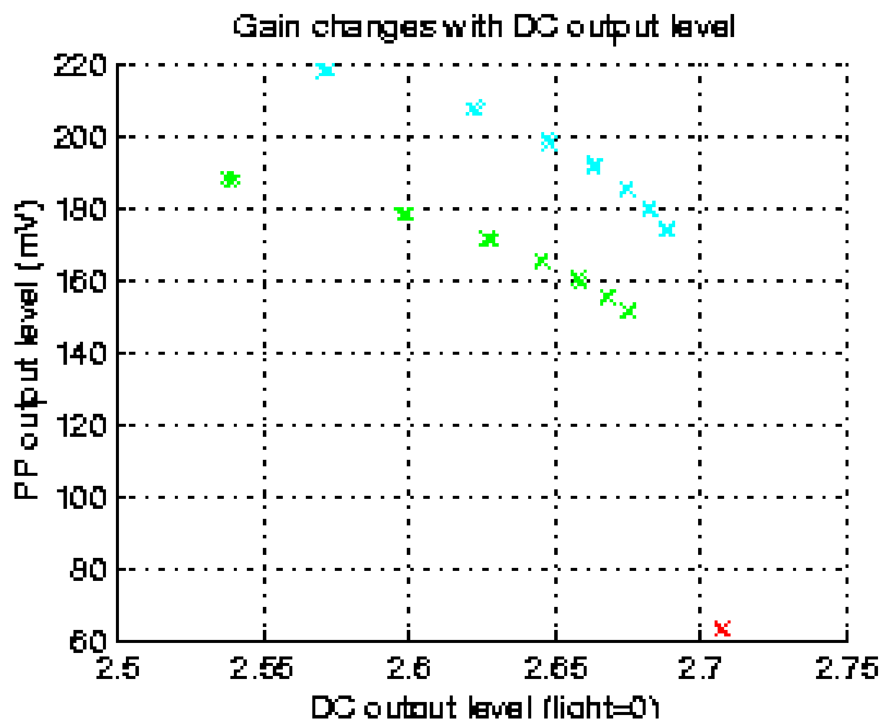
Photoreceptor: effect on AC gain

The gain is defined as:

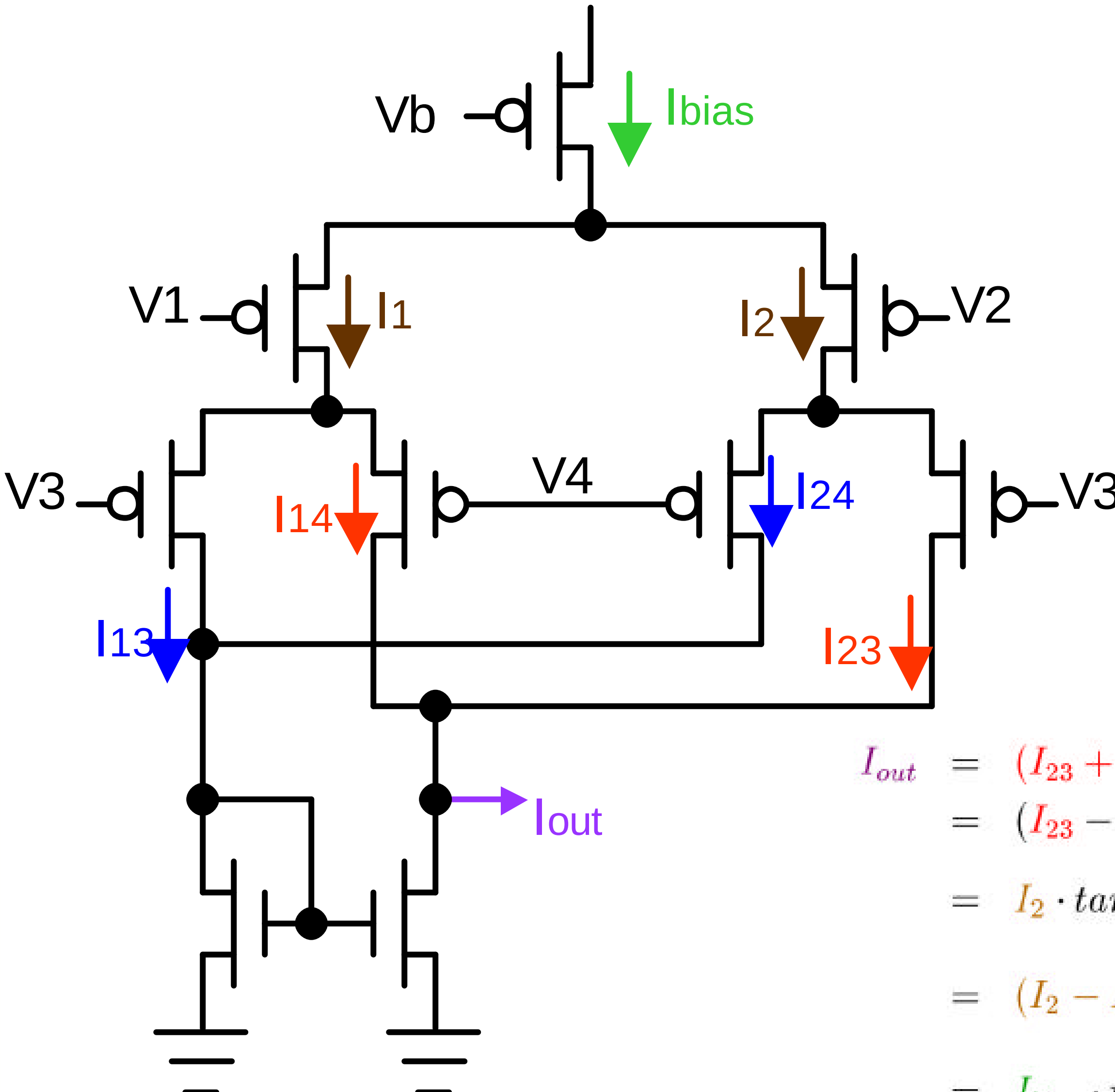
max output (no light) - min output (max light)

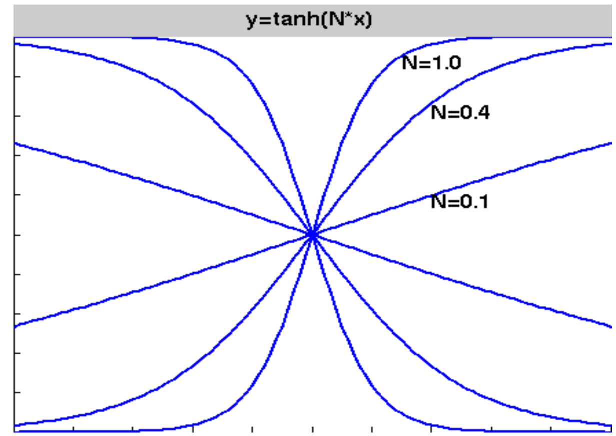
The gain varies slightly as a function of the output DC voltage.





Gilbert multiplier



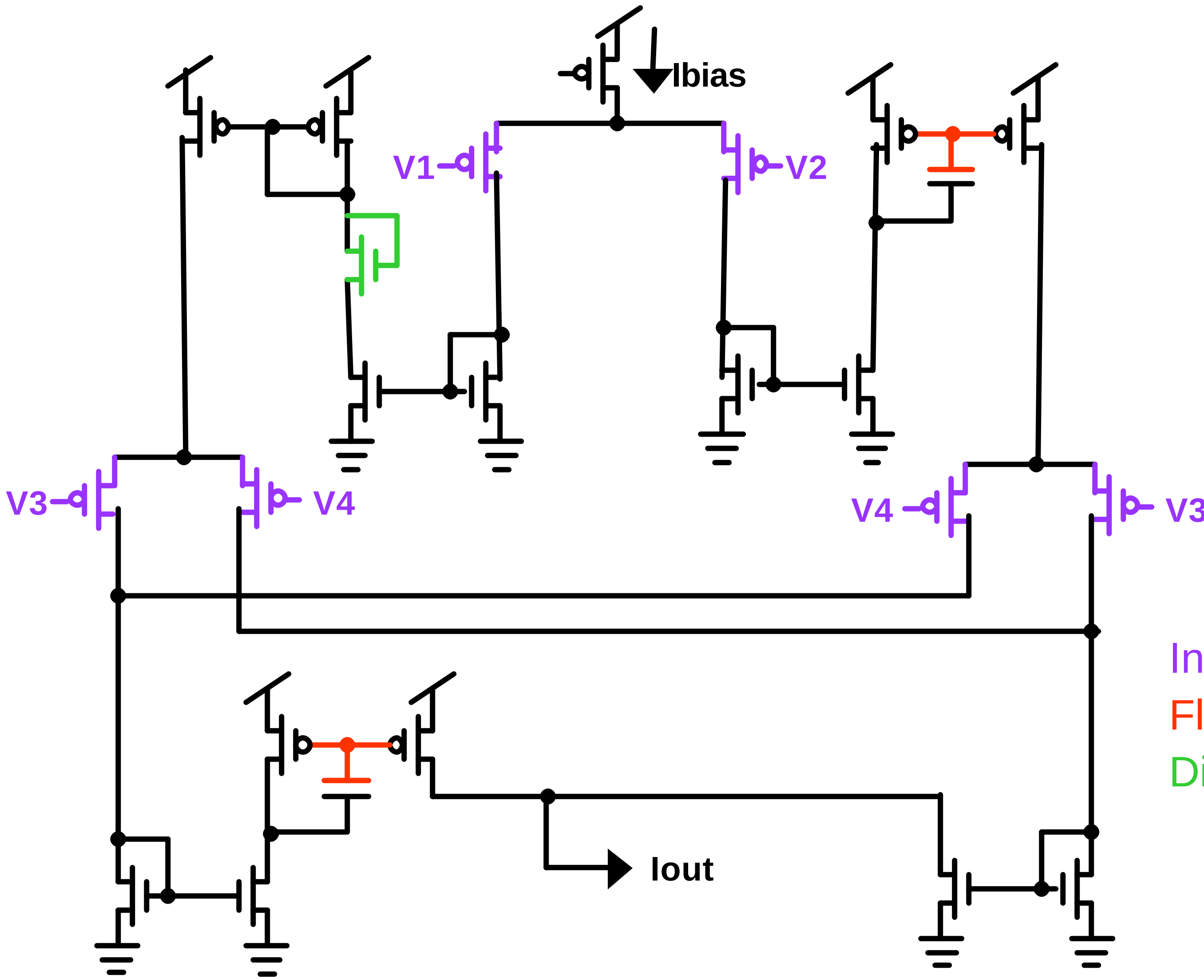


$$
\begin{aligned}
I_{out} &= (I_{23} + I_{14}) - (I_{24} + I_{13}) \\
&= (I_{23} - I_{24}) - (-I_{14} + I_{13}) \\
&= I_2 \cdot tanh\frac{\kappa(V_3 - V_4)}{2} - I_1 \cdot tanh\frac{\kappa(V_3 - V_4)}{2} \\
&= (I_2 - I_1) \cdot tanh\frac{\kappa(V_3 - V_4)}{2} \\
&= I_{bias} \cdot tanh\frac{\kappa(V_2 - V_1)}{2} \cdot tanh\frac{\kappa(V_3 - V_4)}{2}
\end{aligned}
$$

Problems with Gilbert

- Limited range: Must keep all transistors in saturation; distortion
  - Limited output range: Vout < Vmax = K(max(V3,V4)-Vb)
  - Limited input range: similar constraint for inputs V1 and V2
- No good place to put floating gates for adaptation:
  - FG on inputs V1 or V2 results in gain mismatch between inputs
  - Since V3 and V4 go to two transistors each, we would have to have two floating gates to remove offsets between V3 and V4. Total: 3 floating gates are too many.
- Solution: use current mirrors to isolate the V1-V2 diff pairs from V3-V4 pairs. Increase headroom.

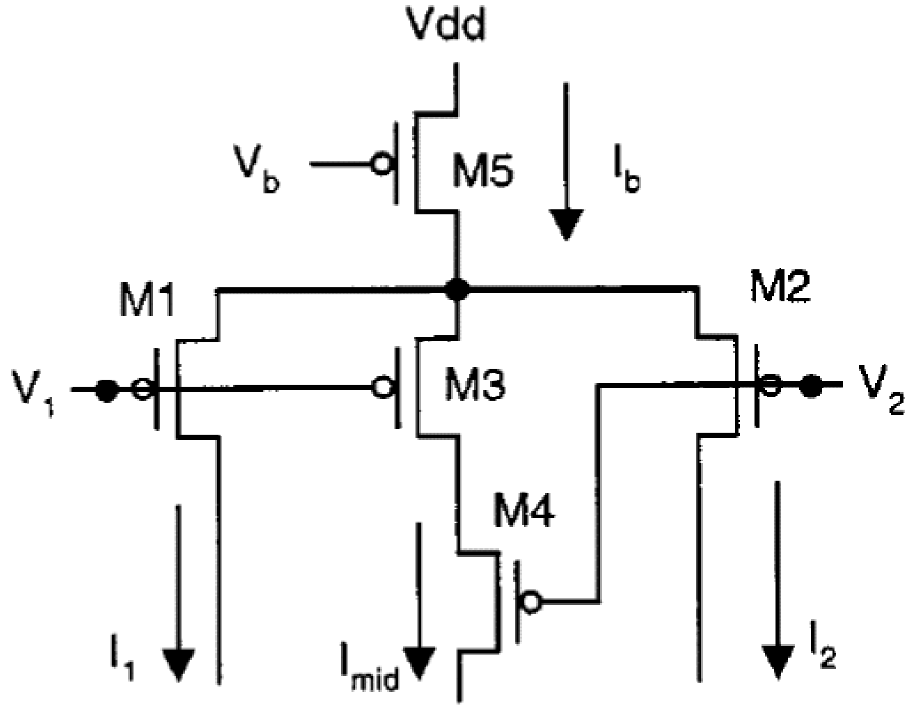
Revised multiplier



Inputs
Floating gates
Diode (V shift)

Bump circuit

- Computes similarity between inputs V1 and V2
- I_{mid} shaped like a Gaussian



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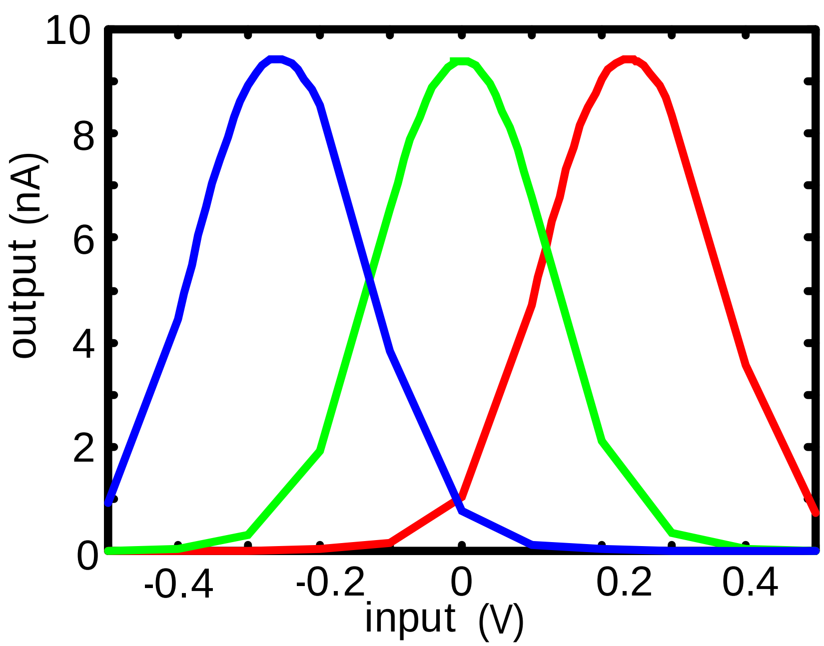
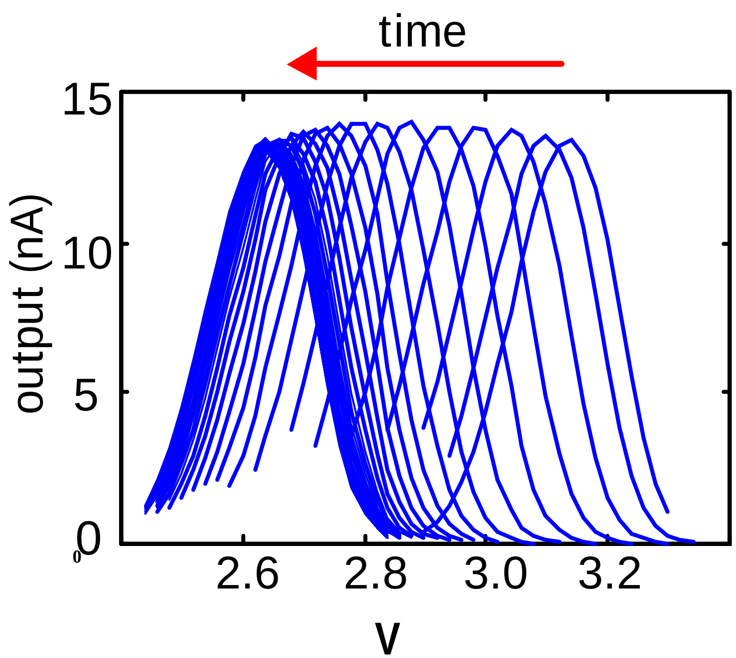
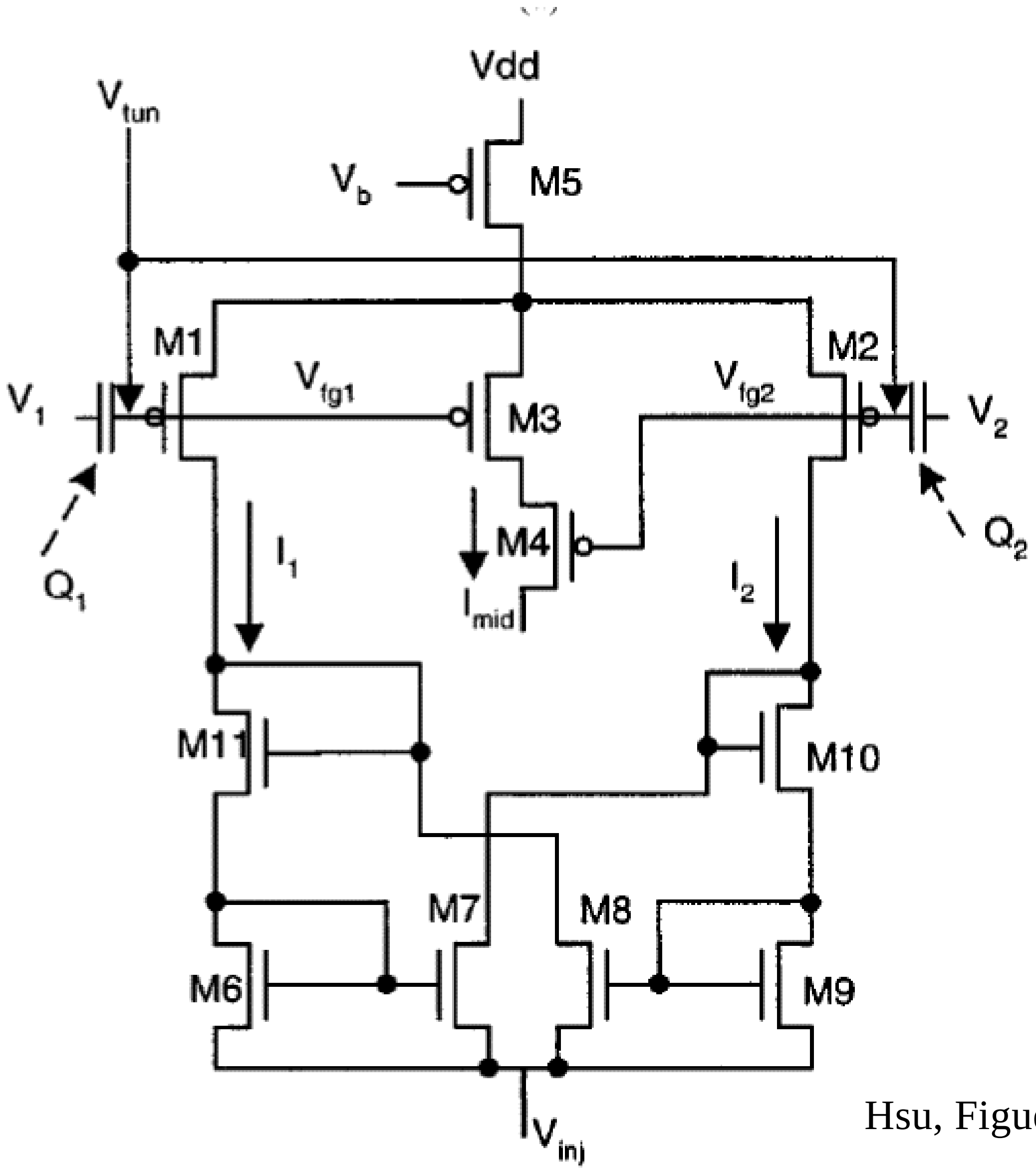
Adaptive bump circuit

- Bump circuit is a 1-D Gaussian distribution in silicon
  - Stores nonvolatile analog value μ
  - Computes Gaussian-like probability, P(x|μ)
  - Adapts μ to increase P(x|μ)
  - In 14 transistors



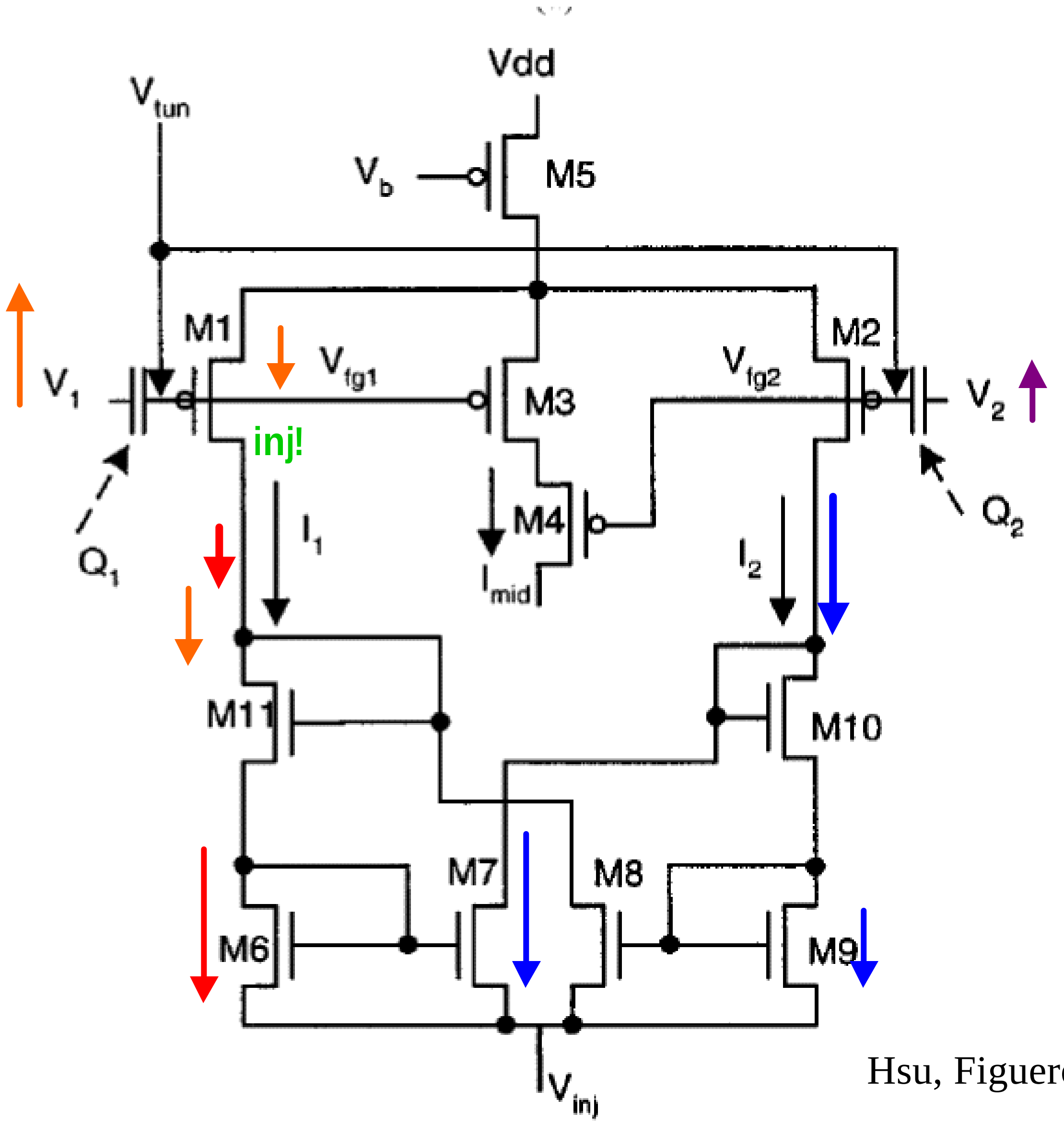


Adaptive bump circuit



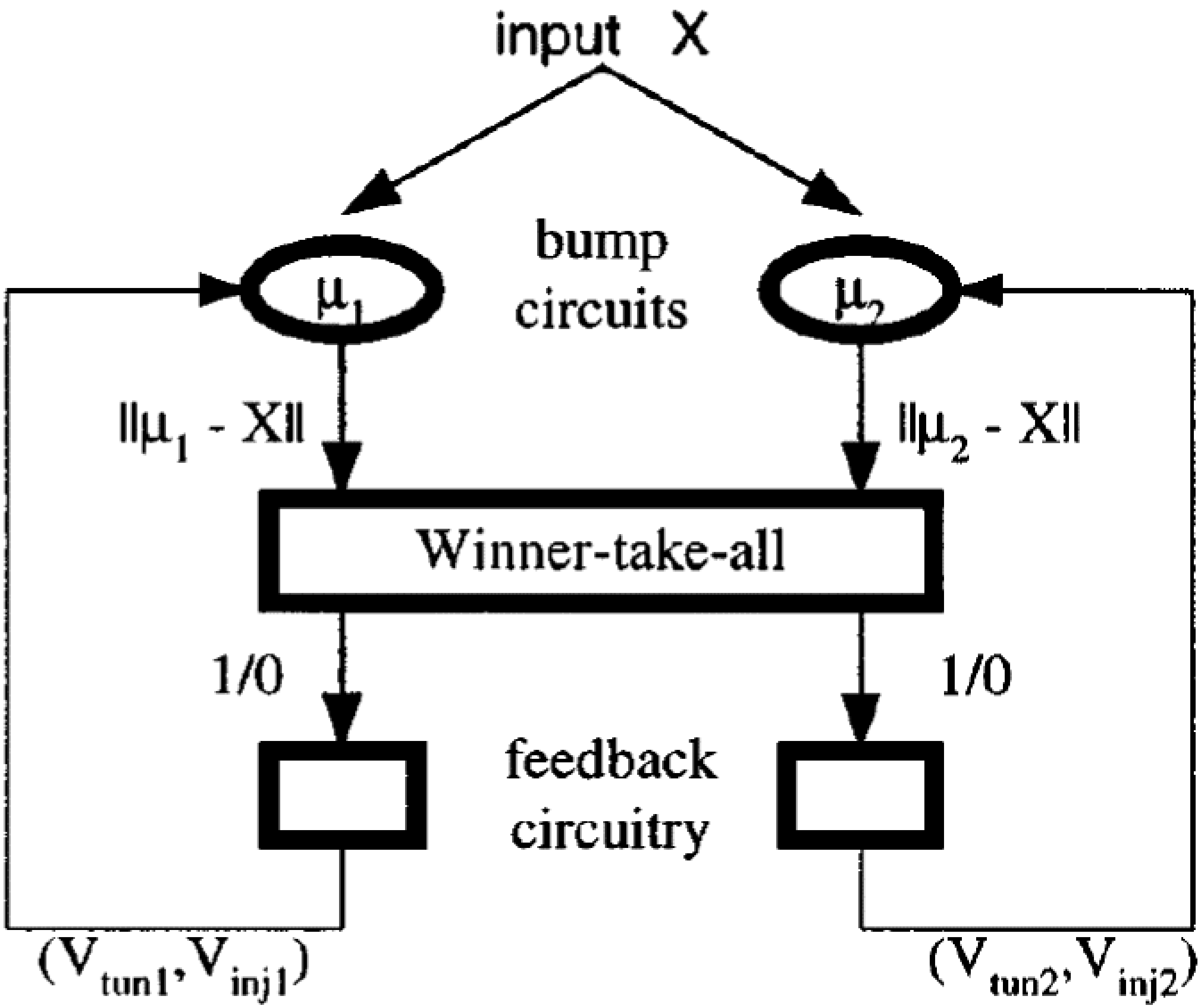
Hsu, Figueroa, Diorio 2002

Adaptive bump circuit

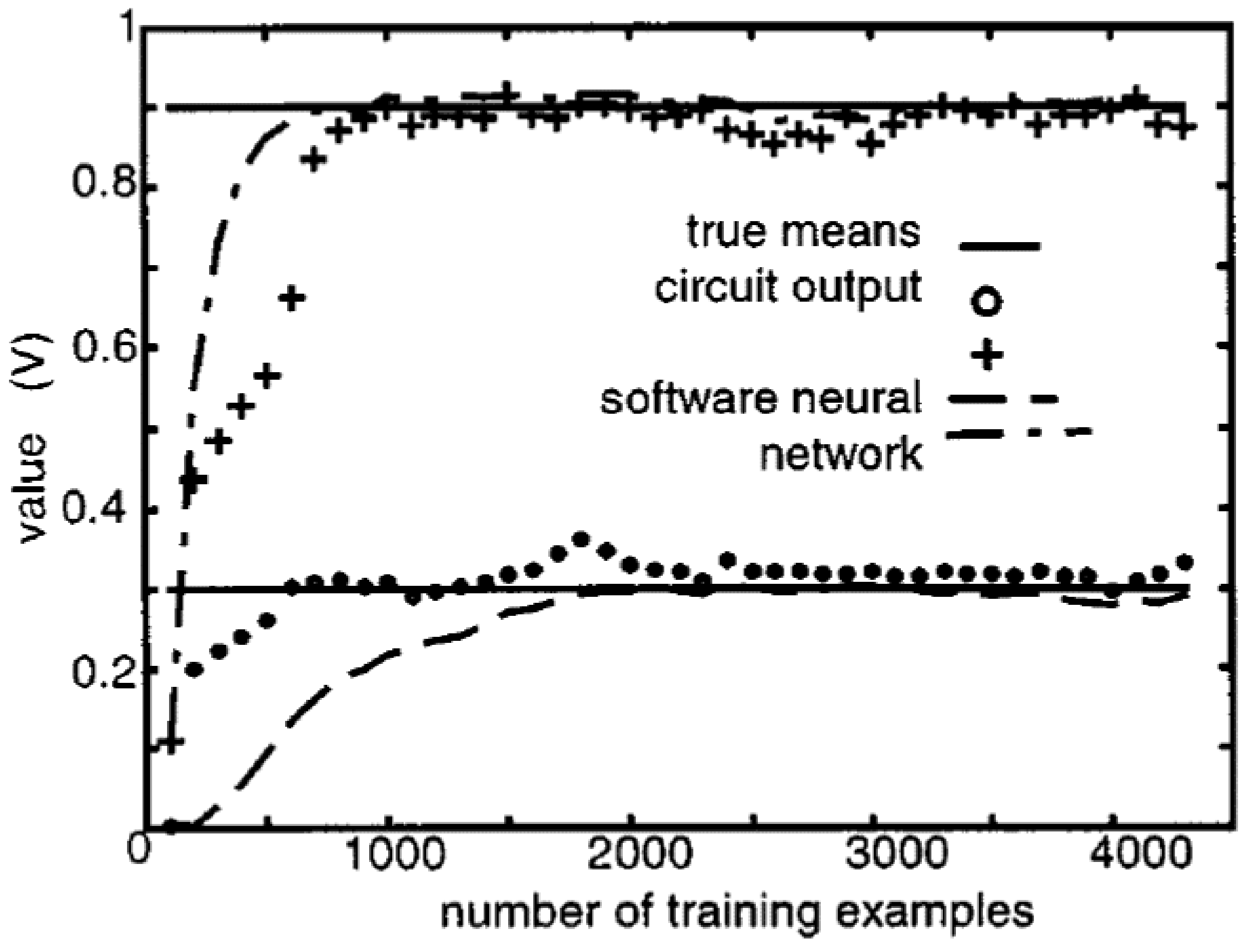


Hsu, Figueroa, Diorio 2002

Learning Mixture of Gaussians



Mixture of Gaussians data



David Hsu

The end

Appendices follow

Photoreceptor equations

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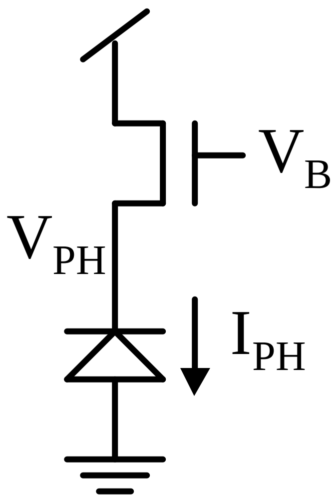
The very basic logarithmic photoreceptor:

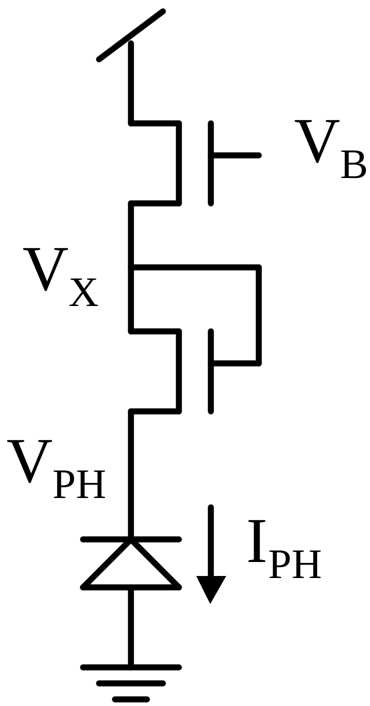


$$I_{PH} = I_O \exp(V_B - V_{PH}) = I_O^{'} \exp(-V_{PH})$$

$$V_{PH} = -\log\left(\frac{I_{PH}}{I_O^{'}}\right)$$

Photoreceptor equations

Double the impedance, double the gain:



$$I_{PH} = I_O \exp(V_B - V_X) = I_O \exp(V_X - V_{PH})$$

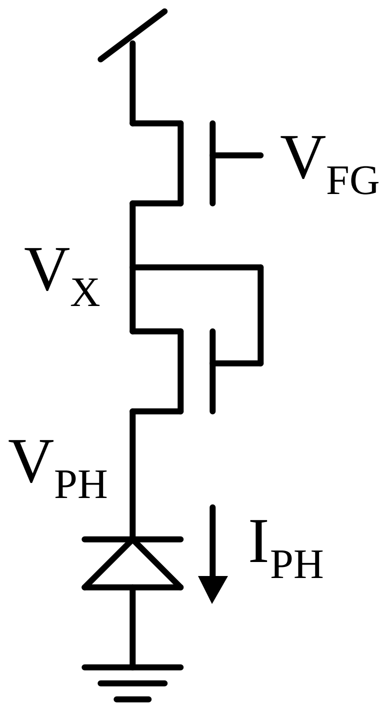
$$V_X = \frac{V_B + V_{PH}}{2}$$

$$I_{PH} = I_O \exp\left(\frac{V_B}{2} - \frac{V_{PH}}{2}\right) = I_O^{'} \exp\left(-\frac{V_{PH}}{2}\right)$$

$$V_{PH} = -2\log\left(\frac{I_{PH}}{I_O^{'}}\right)$$

Photoreceptor equations

Use a floating gate instead of a bias:



$$I_{PH} = I_O \exp(V_{FG} - V_X) = I_O \exp(V_X - V_{PH})$$

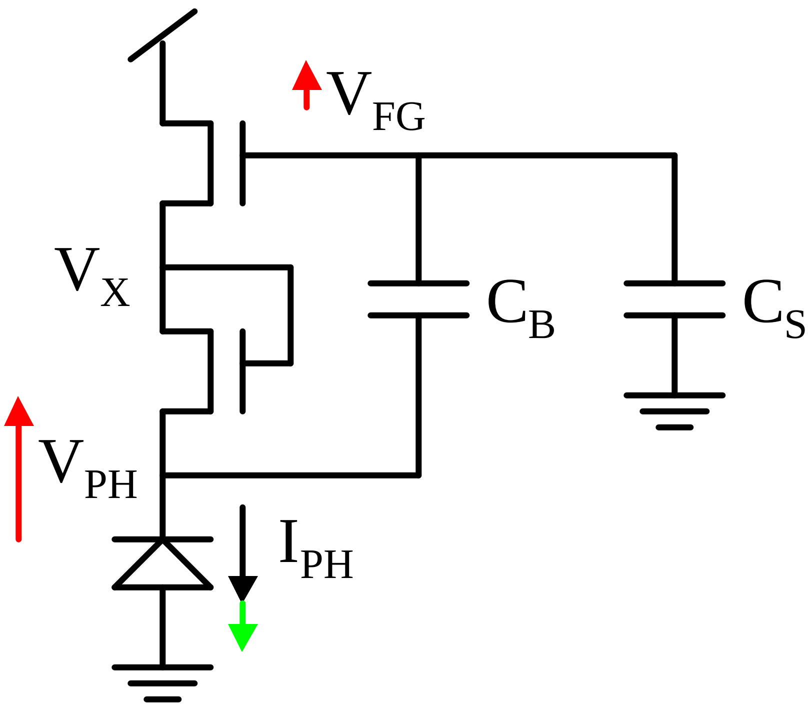
$$V_X = \frac{V_{FG} + V_{PH}}{2}$$

$$I_{PH} = I_O \exp\left(\frac{V_{FG}}{2} - \frac{V_{PH}}{2}\right)$$

$$V_{PH} = -2\log\left(\frac{I_{PH}}{I_O}\right) + V_{FG}$$

Photoreceptor equations

Add AC gain:



$$V_{FG} = \frac{C_B}{C_S + C_B} V_{PH}$$

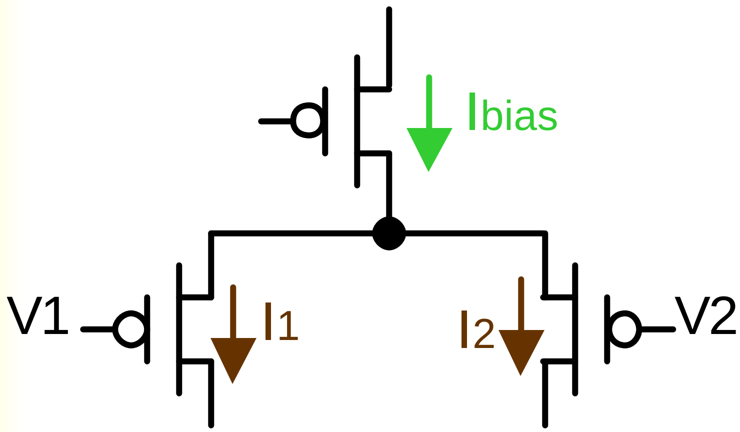
$$V_{PH} = -2\log\left(\frac{I_{PH}}{I_O}\right) + V_{FG}$$

$$V_{PH} = -2\log\left(\frac{I_{PH}}{I_O}\right) + \frac{C_B}{C_S + C_B} V_{PH}$$

$$V_{PH} = \frac{-2\log\left(\frac{I_{PH}}{I_O}\right)}{1 - \frac{C_B}{C_S + C_B}}$$

$$V_{PH} = -2\left(\frac{C_S + C_B}{C_S}\right)\log\left(\frac{I_{PH}}{I_O}\right)$$

Differential Pair



voltage-current relation in subthreshold:

$$
\begin{aligned}
I_1 &= I_o e^{(V_s - \kappa V_1)} \\
I_2 &= I_o e^{(V_s - \kappa V_2)} \\
I_{bias} &= I_1 + I_2 \\
&= I_o e^{-V_s} \left(e^{\kappa V_1} + e^{\kappa V_2} \right)
\end{aligned}
$$

current split between I_1 and I_2:

$$
\begin{aligned}
I_1 &= I_{bias} \frac{e^{\kappa V_1}}{e^{\kappa V_1} + e^{\kappa V_2}} \\
I_2 &= I_{bias} \frac{e^{\kappa V_2}}{e^{\kappa V_1} + e^{\kappa V_2}}
\end{aligned}
$$

$$
I_1 - I_2 = I_{bias} \frac{e^{\kappa V_1} - e^{\kappa V_2}}{e^{\kappa V_1} + e^{\kappa V_2}}
$$

multiply by $e^{-(V_1+V_2)/2}$

$$
I_1 - I_2 = I_{bias} \frac{e^{\kappa (V_1 - V_2)/2} - e^{\kappa (V_2 - V_1)/2}}{e^{\kappa (V_1 - V_2)/2} + e^{\kappa (V_2 - V_1)/2}} = I_{bias} tanh \frac{\kappa (V_1 - V_2)}{2}
$$