

Outline

Introduction

Approximate inference

Derivation of game

Definition of game

Scores for comparing approximations

Theoretical results

Experimental results

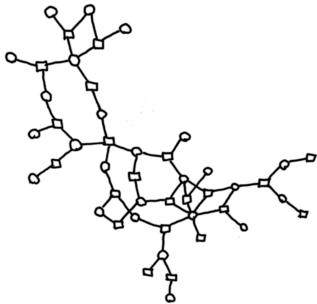
Conclusion

A conditional game for comparing approximations

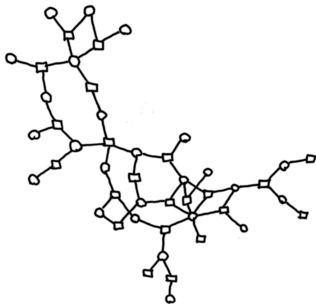
Frederik Eaton
Computational and Biological Learning Laboratory
University of Cambridge

April 11, 2011

model →



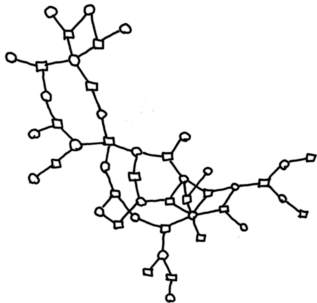
model \rightarrow



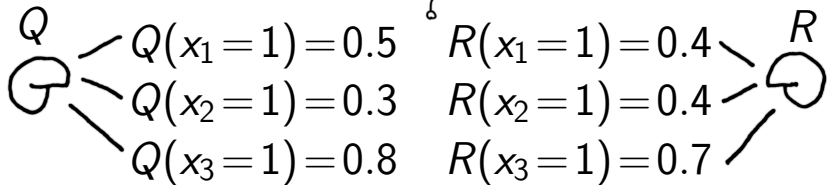
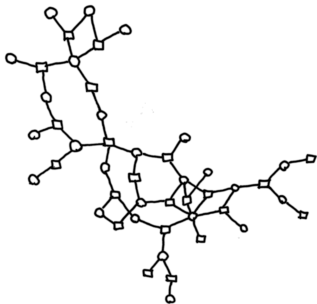
approximate inference
algorithm \rightarrow

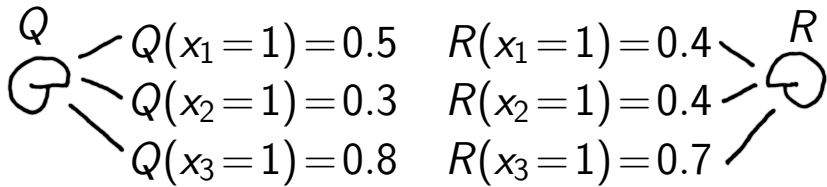


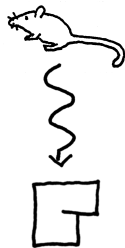
\mathbb{Q}



\mathbb{R}







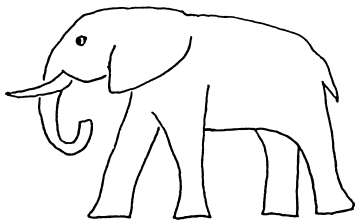


???



???





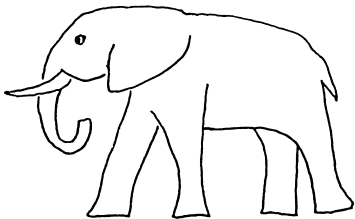
Q

?

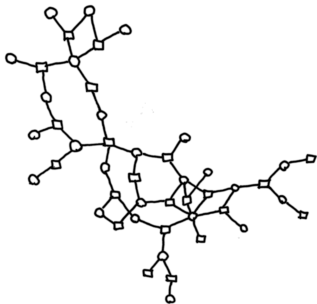


?

R

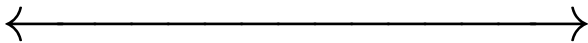


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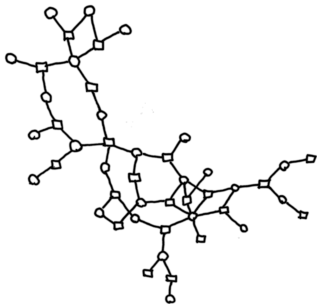


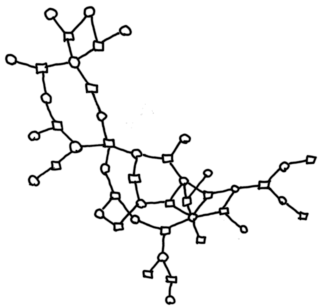
\mathbb{R}

\mathbb{Q}

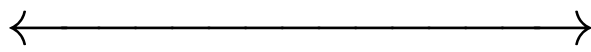


\mathbb{R}



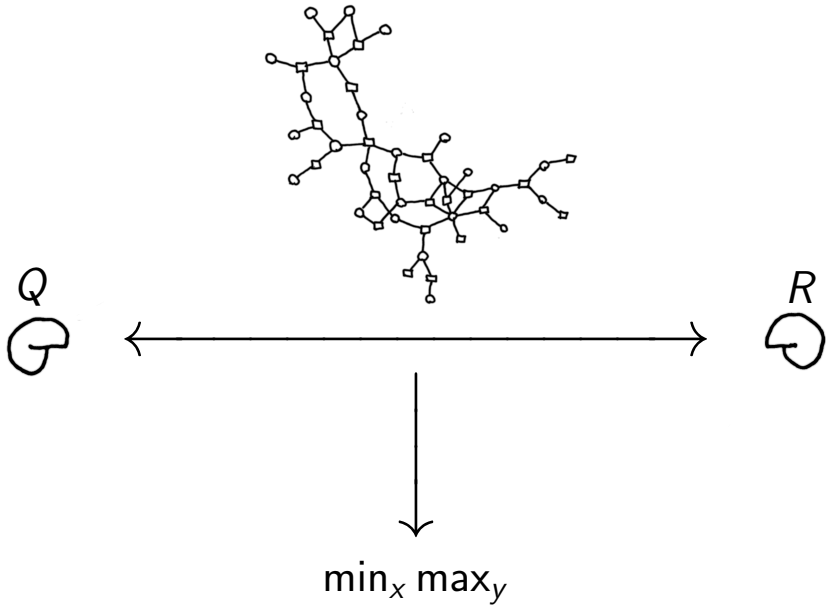


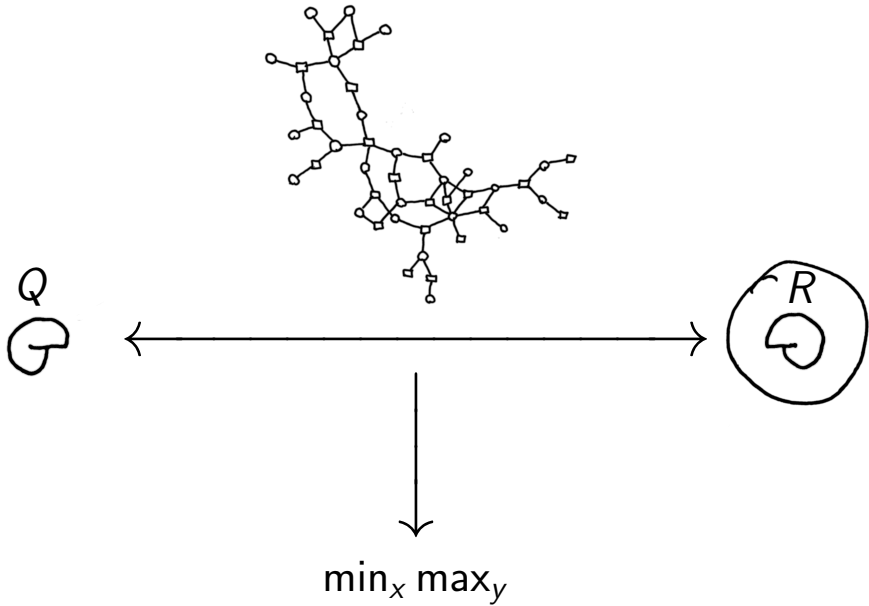
Q
 G



R
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Overview

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- ▶ The conditional game can compare approximations for accuracy

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- ▶ First way to do this

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- ▶ First way to do this
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- ▶ Empirical investigations
- ▶ Some pros, some cons. Future work

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Approximate Inference

random state $x = (x_1, \dots, x_i, \dots, x_n)$

Approximate Inference

random state $x = (x_1, \dots, x_i, \dots, x_n)$, factors α

$$\prod_{\alpha} \psi_{\alpha}(x_{\alpha})$$

Approximate Inference

random state $x = (x_1, \dots, x_i, \dots, x_n)$, factors α

$$P(x) = \frac{1}{Z} \prod_{\alpha} \psi_{\alpha}(x_{\alpha})$$

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$$\sum_{x_{\setminus i}} P(x)$$

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$$\sum_{x_{\setminus i}} P(x) = P(x_i)$$

Approximate Inference

random state $x = (x_1, \dots, x_i, \dots, x_n)$, factors α

$$P(x) = \frac{1}{Z} \prod_{\alpha} \psi_{\alpha}(x_{\alpha})$$

$$(Z = \sum_x \prod_{\alpha} \psi_{\alpha}(x_{\alpha}))$$

$$Q(x_i) \approx \sum_{x_{\setminus i}} P(x) = P(x_i)$$

approximate inference

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exact inference

Approximate Inference

random state $\mathbf{x} = (x_1, \dots, x_i, \dots, x_n)$, factors α

$$\delta(\mathbf{x}_k, \mathbf{x}_k^*) \prod_{\alpha} \psi_{\alpha}(\mathbf{x}_{\alpha})$$

Approximate Inference

random state $x = (x_1, \dots, x_i, \dots, x_n)$, factors α

$$P'(x) = \frac{1}{Z'} \delta(x_k, x_k^*) \prod_{\alpha} \psi_{\alpha}(x_{\alpha})$$

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conditioned model

Approximate Inference

random state $x = (x_1, \dots, x_i, \dots, x_n)$, factors α

$$P(x|x_k = x_k^*) = P'(x) = \frac{1}{Z'} \delta(x_k, x_k^*) \prod_{\alpha} \psi_{\alpha}(x_{\alpha})$$
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$$Q(x_i) \approx \sum_{x_{\setminus i}} P'(x)$$

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$$Q(x_i) \approx \sum_{x_{\setminus i}} P'(x) = P(x_i|x_k = x_k^*)$$

conditioned model

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Game requirements

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1. No extra computation

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$$\prod_{\alpha} \psi_{\alpha}(\mathbf{x}_{\alpha})$$

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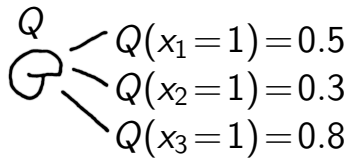
$$ZP(x) = \prod_{\alpha} \psi_{\alpha}(x_{\alpha})$$

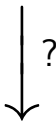
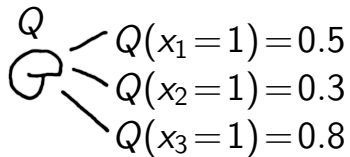
Game requirements

1. No extra computation

$$ZP(x) = \prod_{\alpha} \psi_{\alpha}(x_{\alpha})$$

2. No additional randomness





$$(x_1, x_2, x_3) = (1, 0, 1)$$

Sampling a point

Sampling a point

x_1	x_2	x_3
?		

$$Q(x_1 = 1) = 0.5$$

Sampling a point

x_1	x_2	x_3
?		
1	?	

$$Q(x_1 = 1) = 0.5$$

$$Q(x_2 = 0 | x_1 = 1) = 0.2$$

Sampling a point

x_1	x_2	x_3
?		
1	?	
1	0	?

$$Q(x_1 = 1) = 0.5$$

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$$Q(x_3 = 1 | x_1 = 1, x_2 = 0) = 0.7$$

Sampling a point

x_1	x_2	x_3
?		
1	?	
1	0	?
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$$\longrightarrow ZP(x) = \prod_{\alpha} \psi_{\alpha}(x_1 = 1, x_2 = 0, x_3 = 1)$$

Sampling a point

x_1	x_2	x_3
?		
1	?	
1	0	?
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$$Q(x_1 = 1) = 0.5$$

$$Q(x_2 = 0 | x_1 = 1) = 0.2$$

$$Q(x_3 = 1 | x_1 = 1, x_2 = 0) = 0.7$$

$$\begin{aligned} \longrightarrow Q(x) &= Q(x_1 = 1) \\ &\times Q(x_2 = 0 | x_1 = 1) \\ &\times Q(x_3 = 1 | x_1 = 1, x_2 = 0) \end{aligned}$$

Two quantities

$$Q(\mathbf{x}) = \prod_i Q(\mathbf{x}_i | \mathbf{x}_{1:i-1})$$
$$ZP(\mathbf{x}) = \prod_{\alpha} \psi_{\alpha}(\mathbf{x}_{\alpha})$$

Two quantities

$$\frac{Q(\mathbf{x}) = \prod_i Q(\mathbf{x}_i | \mathbf{x}_{1:i-1})}{ZP(\mathbf{x}) = \prod_{\alpha} \psi_{\alpha}(\mathbf{x}_{\alpha})}$$

Two quantities

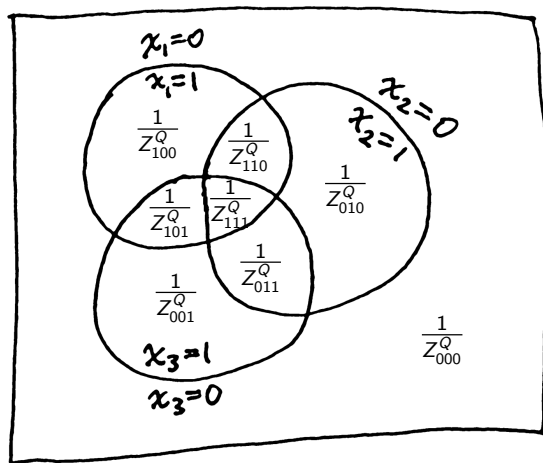
$$\frac{Q(\mathbf{x}) = \prod_i Q(\mathbf{x}_i | \mathbf{x}_{1:i-1})}{ZP(\mathbf{x}) = \prod_{\alpha} \psi_{\alpha}(\mathbf{x}_{\alpha})} \approx \frac{1}{Z}$$

Two quantities

$$\frac{Q(\mathbf{x}) = \prod_i Q(x_i | \mathbf{x}_{1:i-1})}{ZP(\mathbf{x}) = \prod_\alpha \psi_\alpha(\mathbf{x}_\alpha)} = \frac{1}{Z_x^Q}$$

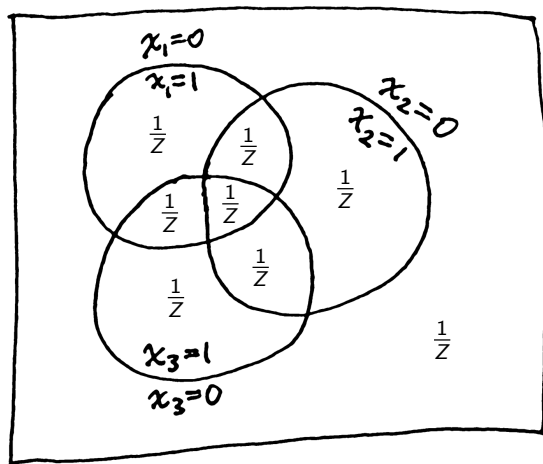
Partition function estimate

$$\frac{1}{Z_x^Q} = \frac{Q(x)}{ZP(x)}$$



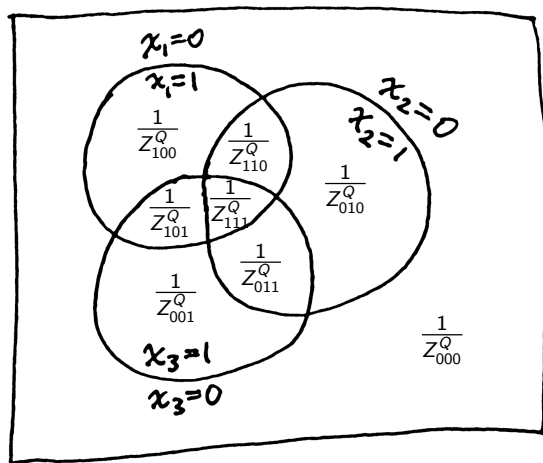
Partition function estimate

$$\frac{1}{Z_x^Q} = \frac{P(x)}{ZP(x)} = \frac{1}{Z}$$



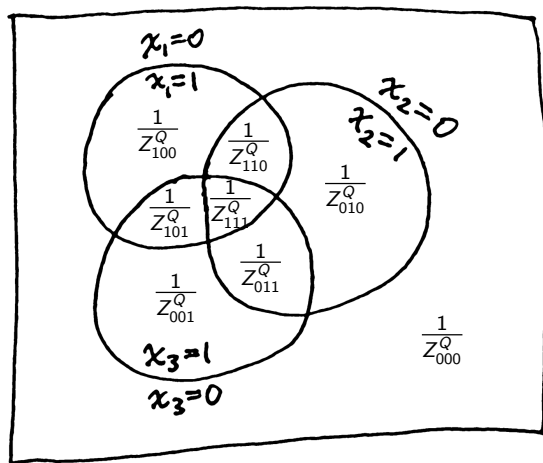
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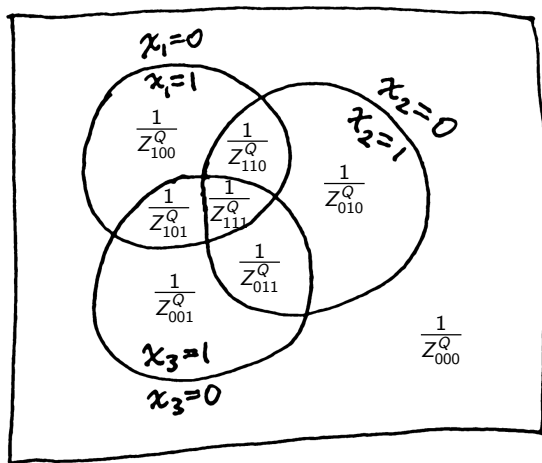
Partition function estimate

$$\frac{1}{Z_x^Q} ZP(x) = Q(x)$$



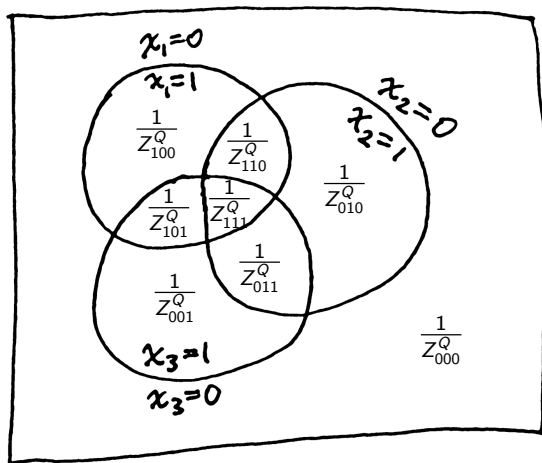
Partition function estimate

$$\sum_x \left(\frac{1}{Z_x^Q} Z P(x) = Q(x) \right) = 1$$



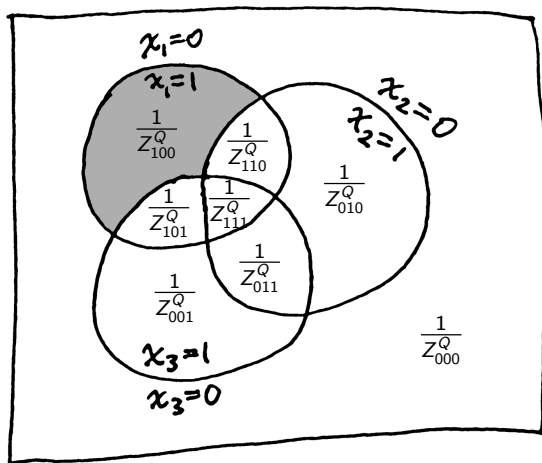
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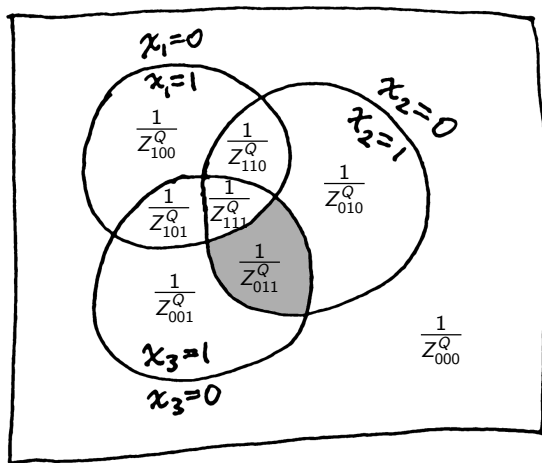
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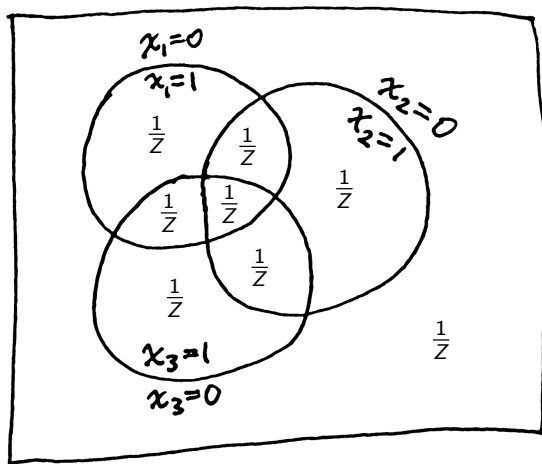
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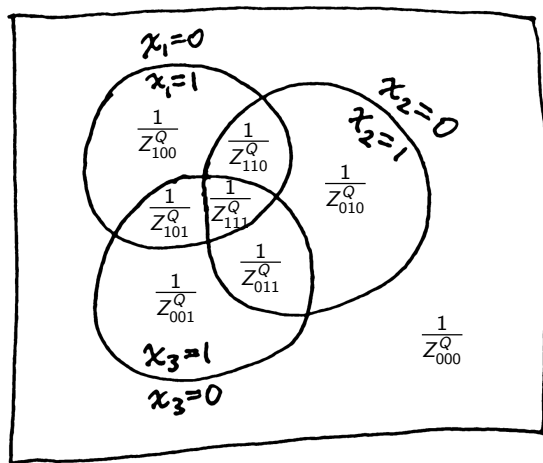
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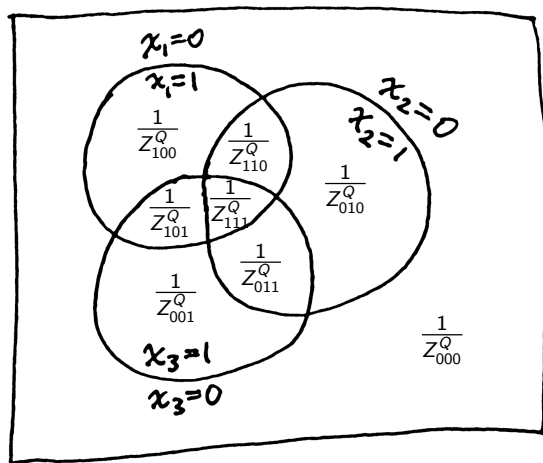
The opponent's strategy

$$Q(x_1) = \sum_{x_{\setminus 1}} \left(Q(x) = \frac{1}{Z_x^Q} ZP(x) \right)$$



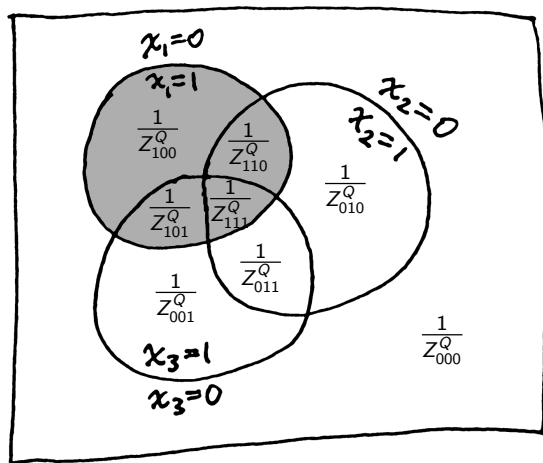
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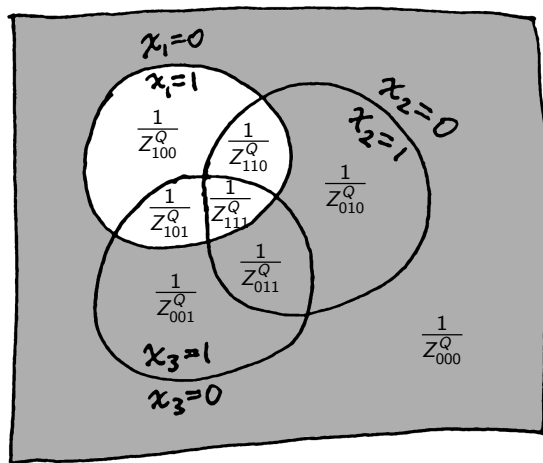
The opponent's strategy

$$P(x_1 = 1) < Q(x_1 = 1) = \sum_{x_{\setminus 1}} \left(Q(x) = \frac{1}{Z_x^Q} \frac{ZP(x)}{Z_x^Q} \right)$$



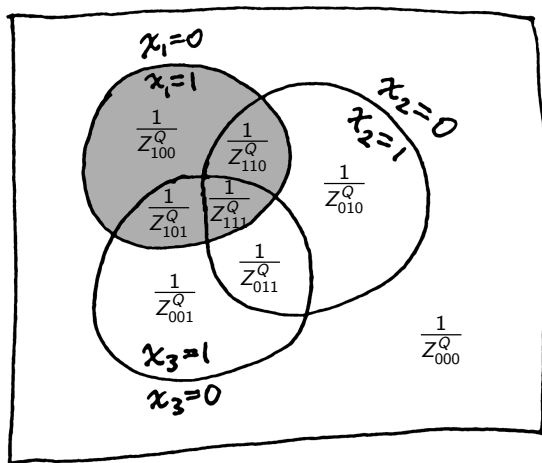
The opponent's strategy

$$P(x_1 = 0) < Q(x_1 = 0) = \sum_{x_{\setminus 1}} \left(Q(x) = \frac{1}{Z_x^Q} \frac{ZP(x)}{Z_x^Q} \right)$$



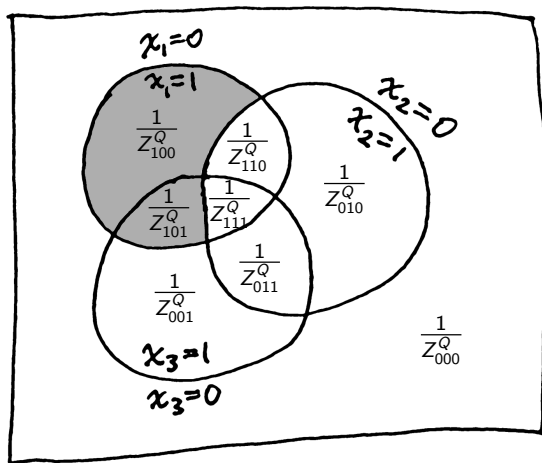
The opponent's strategy

$$R(x_1) \geq \sum_{x \setminus 1} \left(Q(x) = \frac{1}{Z_x^Q} ZP(x) \right)$$



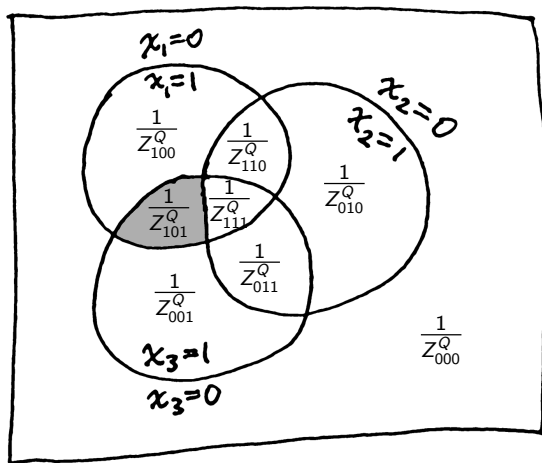
The opponent's strategy

$$R(x_2, x_1) \geq \sum_{x_{1,2}} \left(Q(x) = \frac{1}{Z_x^Q} ZP(x) \right)$$



The opponent's strategy


$$R(x_3, x_1, x_2) \geq Q(x) = \frac{1}{Z_x^Q} ZP(x)$$




The opponent's strategy, restated

$$\frac{Q(x)}{ZP(x)} = \frac{1}{Z_x^Q}$$


The opponent's strategy, restated


$$\frac{Q(x)}{ZP(x)} = \frac{1}{Z_x^Q}$$
$$\frac{Q(x_1) \times Q(x_2|x_1) \times Q(x_3|x_1, x_2)}{ZP(x)}$$


The opponent's strategy, restated


$$\frac{Q(x)}{ZP(x)} = \frac{1}{Z_x^Q}$$
$$\frac{Q(x_1) \times Q(x_2|x_1) \times Q(x_3|x_1, x_2)}{ZP(x)} \geq R(x_1)$$

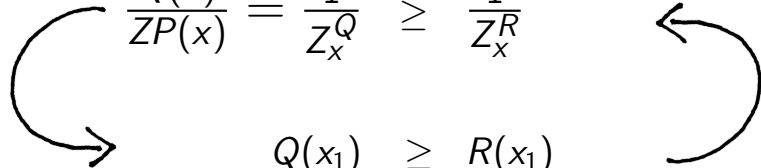
The opponent's strategy, restated


$$\frac{Q(x)}{ZP(x)} = \frac{1}{Z_x^Q}$$
$$\frac{Q(x_1) \times Q(x_2|x_1) \times Q(x_3|x_1, x_2)}{ZP(x)} \geq R(x_1) \geq R(x_2|x_1)$$

The opponent's strategy, restated


$$\frac{Q(x)}{ZP(x)} = \frac{1}{Z_x^Q}$$
$$\frac{\begin{aligned} &Q(x_1) &&\geq &&R(x_1) \\ &\times Q(x_2|x_1) &&\geq &&R(x_2|x_1) \\ &\times Q(x_3|x_1, x_2) &&\geq &&R(x_3|x_1, x_2) \\ \hline &ZP(x) \end{aligned}}{\quad}$$

The opponent's strategy, restated

$$\frac{Q(x)}{ZP(x)} = \frac{1}{Z_x^Q} \geq \frac{1}{Z_x^R}$$

$$\frac{\begin{aligned} &Q(x_1) \\ &\times Q(x_2|x_1) \\ &\times Q(x_3|x_1, x_2) \end{aligned}}{ZP(x)} \geq \begin{aligned} &R(x_1) \\ &R(x_2|x_1) \\ &R(x_3|x_1, x_2) \end{aligned}$$

Strategies versus conditioned marginals

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Strategies versus conditioned marginals

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- ▶ In general, strategies \supset approximations
- ▶ Possible to simulate opponent
- ▶ Unfold game tree for several moves
- ▶ \implies Too expensive
- ▶ Better to pretend opponent unknown, second-guessing impossible

Strategies versus conditioned marginals

- ▶ Conditioned marginal approximations define players
- ▶ In general, strategies \supset approximations
- ▶ Possible to simulate opponent
- ▶ Unfold game tree for several moves
- ▶ \implies Too expensive
- ▶ Better to pretend opponent unknown, second-guessing impossible
- ▶ Assume players use natural strategies

Outline

Introduction

Approximate inference

Derivation of game

Definition of game

Scores for comparing approximations

Theoretical results

Experimental results

Conclusion

Definition of conditional game

Marginal player

Definition of conditional game

Marginal player, conditional player

Definition of conditional game

Marginal player, conditional player (Q, R)

Definition of conditional game

Marginal player, conditional player (Q, R)

Turn $i \in 1 \dots n$:

Definition of conditional game

Marginal player, conditional player (Q, R)

Turn $i \in 1 \dots n$:

- ▶ MP: advertises marginals $Q(x_i | x_{1:i-1}^*)$

Definition of conditional game

Marginal player, conditional player (Q, R)

Turn $i \in 1 \dots n$:

- ▶ MP: advertises marginals $Q(x_i | x_{1:i-1}^*)$
- ▶ CP: chooses a state x_i^*

Definition of conditional game

Marginal player, conditional player (Q, R)

Turn $i \in 1 \dots n$:

- ▶ MP: advertises marginals $Q(x_i | x_{1:i-1}^*)$
- ▶ CP: chooses a state x_i^*

$$x_i^* = \arg \max_{x_i} \frac{Q(x_i | x_{1:i-1}^*)}{R(x_i | x_{1:i-1}^*)}$$

Definition of conditional game

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$(\rightarrow x^*, \{Q(x_i^* | x_{1:i-1}^*)\}_i)$

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Value of game:

$$V^+(Q, R) \equiv \log \frac{1}{Z_{x^*}^Q} = \log \frac{\prod_i Q(x_i^* | x_{1:i-1}^*)}{\prod_\alpha \psi_\alpha(x_\alpha^*)}$$

Definition of conditional game

Marginal player, conditional player (Q, R)

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Definition of conditional game

Marginal player, conditional player (Q, R)

Turn $i \in 1 \dots n$:

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$$x_i^* = \underset{x_i}{\text{arg min}} \frac{Q(x_i | x_{1:i-1}^*)}{R(x_i | x_{1:i-1}^*)}$$

$(\rightarrow x^*, \{Q(x_i^* | x_{1:i-1}^*)\}_i)$

Value of game:

$$V^-(Q, R) \equiv \log \frac{1}{Z_{x^*}^Q} = \log \frac{\prod_i Q(x_i^* | x_{1:i-1}^*)}{\prod_\alpha \psi_\alpha(x_\alpha^*)}$$

Variable order

Turn $i \in 1 \dots n$:

Value:

CP chooses:

$$V^+(Q, R)$$

$$\arg \max_{x_i} \frac{Q(x_i | x_{1:i-1}^*)}{R(x_i | x_{1:i-1}^*)}$$

$$V^-(Q, R)$$

$$\arg \min_{x_i} \frac{Q(x_i | x_{1:i-1}^*)}{R(x_i | x_{1:i-1}^*)}$$

$$V^+ = V^- = \log \frac{1}{Z_x^Q}$$

Variable order

Turn $t \in 1 \dots n$:

Value:

CP chooses:

$$V^+(Q, R)$$

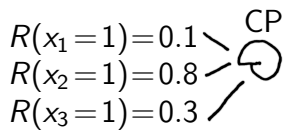
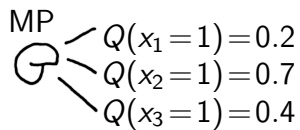
$$\arg \max_{(i_t \notin i_{1:t-1}, x_{i_t})} \frac{Q(x_{i_t} | x_{i_{1:t-1}}^*)}{R(x_{i_t} | x_{i_{1:t-1}}^*)}$$

$$V^-(Q, R)$$

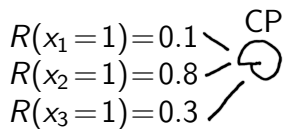
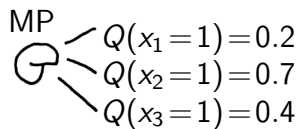
$$\arg \min_{(i_t \notin i_{1:t-1}, x_{i_t})} \frac{Q(x_{i_t} | x_{i_{1:t-1}}^*)}{R(x_{i_t} | x_{i_{1:t-1}}^*)}$$

$$V^+ = V^- = \log \frac{1}{Z_x^Q}$$

An example game



An example game



$x_{t_1} x_{t_2} x_{t_3}$

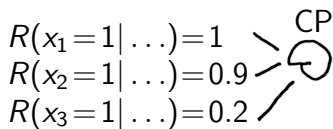
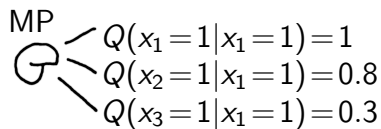
1		
---	--	--

$$t_1 = 1$$

$$x_{t_1}^* = 1$$

$$Q(x_1^*) = 0.2$$

An example game



$x_{t_1} x_{t_2} x_{t_3}$

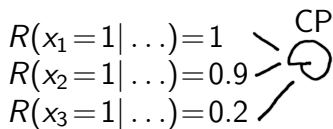
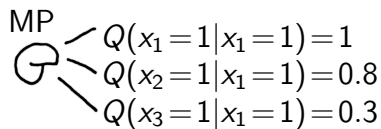
1		
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$$Q(x_1^*) = 0.2$$

An example game



$x_{t_1} x_{t_2} x_{t_3}$

1		
---	--	--

1	0	
---	---	--

$t_1 = 1$

$t_2 = 2$

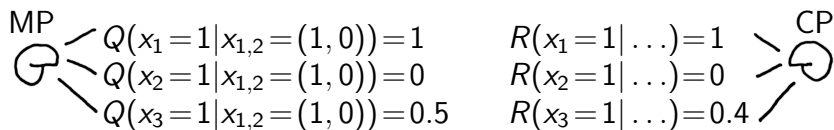
$x_{t_1}^* = 1$

$x_{t_2}^* = 0$

$Q(x_1^*) = 0.2$

$Q(x_2^* | x_1^*) = 0.2$

An example game



$x_{t_1} x_{t_2} x_{t_3}$

1		
---	--	--

1	0	
---	---	--

$t_1 = 1$

$t_2 = 2$

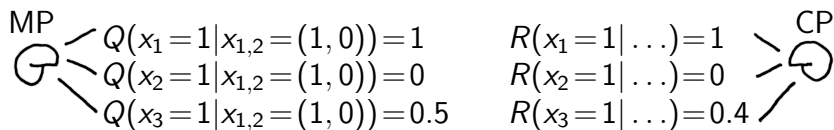
$x_{t_1}^* = 1$

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$Q(x_1^*) = 0.2$

$Q(x_2^* | x_1^*) = 0.2$

An example game



$x_{t_1} x_{t_2} x_{t_3}$

1		
---	--	--

1	0	
---	---	--

1	0	1
---	---	---

$t_1 = 1$

$t_2 = 2$

$t_3 = 3$

$x_{t_1}^* = 1$

$x_{t_2}^* = 0$

$x_{t_3}^* = 1$

$Q(x_1^*) = 0.2$

$Q(x_2^* | x_1^*) = 0.2$

$Q(x_3^* | x_{1,2}^*) = 0.5$

An example game

MP



CP



$x_{t_1} x_{t_2} x_{t_3}$

1		
---	--	--

1	0	
---	---	--

1	0	1
---	---	---

$$t_1 = 1$$

$$x_{t_1}^* = 1$$

$$Q(x_1^*) = 0.2$$

$$t_2 = 2$$

$$x_{t_2}^* = 0$$

$$Q(x_2^* | x_1^*) = 0.2$$

$$t_3 = 3$$

$$x_{t_3}^* = 1$$

$$Q(x_3^* | x_{1,2}^*) = 0.5$$

$$x^* = (1, 0, 1), \quad Q(x^*) = 0.2 \times 0.2 \times 0.5 = 0.02$$

An example game

MP



CP



$x_{t_1} x_{t_2} x_{t_3}$

1		
---	--	--

1	0	
---	---	--

1	0	1
---	---	---

$t_1 = 1$

$x_{t_1}^* = 1$

$Q(x_1^*) = 0.2$

$t_2 = 2$

$x_{t_2}^* = 0$

$Q(x_2^* | x_1^*) = 0.2$

$t_3 = 3$

$x_{t_3}^* = 1$

$Q(x_3^* | x_{1,2}^*) = 0.5$

$$x^* = (1, 0, 1), \quad Q(x^*) = 0.2 \times 0.2 \times 0.5 = 0.02$$

$$V^+(Q, R) = \log \frac{1}{Z_{(1,0,1)}^Q}$$

An example game

MP



CP



$x_{t_1} x_{t_2} x_{t_3}$

1		
---	--	--

$$t_1 = 1$$

$$x_{t_1}^* = 1$$

$$Q(x_1^*) = 0.2$$

1	0	
---	---	--

$$t_2 = 2$$

$$x_{t_2}^* = 0$$

$$Q(x_2^* | x_1^*) = 0.2$$

1	0	1
---	---	---

$$t_3 = 3$$

$$x_{t_3}^* = 1$$

$$Q(x_3^* | x_{1,2}^*) = 0.5$$

$$x^* = (1, 0, 1), \quad Q(x^*) = 0.2 \times 0.2 \times 0.5 = 0.02$$

$$V^+(Q, R) = \log \frac{1}{Z_{(1,0,1)}^Q} = \log \frac{0.2 \times 0.2 \times 0.5}{\prod_{\alpha} \psi_{\alpha}(x_{\alpha}^* = (1, 0, 1))}$$

An example game

MP



CP



$x_{t_1} x_{t_2} x_{t_3}$

1		
---	--	--

1	0	
---	---	--

1	0	1
---	---	---

$t_1 = 1$

$x_{t_1}^* = 1$

$Q(x_1^*) = 0.2$

$t_2 = 2$

$x_{t_2}^* = 0$

$Q(x_2^* | x_1^*) = 0.2$

$t_3 = 3$

$x_{t_3}^* = 1$

$Q(x_3^* | x_{1,2}^*) = 0.5$

$$x^* = (1, 0, 1), \quad Q(x^*) = 0.2 \times 0.2 \times 0.5 = 0.02$$

$$V^+(Q, R) = \log \frac{1}{Z_{(1,0,1)}^Q} = \log \frac{0.2 \times 0.2 \times 0.5}{\prod_{\alpha} \psi_{\alpha}(x_{\alpha}^* = (1, 0, 1))} = \log \frac{0.02}{10}$$

An example game

MP



CP



$x_{t_1} x_{t_2} x_{t_3}$

1		
---	--	--

1	0	
---	---	--

1	0	1
---	---	---

$$t_1 = 1$$

$$t_2 = 2$$

$$t_3 = 3$$

$$x_{t_1}^* = 1$$

$$x_{t_2}^* = 0$$

$$x_{t_3}^* = 1$$

$$Q(x_1^*) = 0.2$$

$$Q(x_2^* | x_1^*) = 0.2$$

$$Q(x_3^* | x_{1,2}^*) = 0.5$$

$$x^* = (1, 0, 1), \quad Q(x^*) = 0.2 \times 0.2 \times 0.5 = 0.02$$

$$V^+(Q, R) = -\log 500$$

An example game

MP



CP



$x_{t_1} x_{t_2} x_{t_3}$

1		
---	--	--

$t_1 = 1$

$x_{t_1}^* = 1$

$Q(x_1^*) = 0.2$

1	0	
---	---	--

$t_2 = 2$

$x_{t_2}^* = 0$

$Q(x_2^* | x_1^*) = 0.2$

1	0	1
---	---	---

$t_3 = 3$

$x_{t_3}^* = 1$

$Q(x_3^* | x_{1,2}^*) = 0.5$

$$x^* = (1, 0, 1), \quad Q(x^*) = 0.2 \times 0.2 \times 0.5 = 0.02$$

$$V^+(Q, R) = -\log 500 \stackrel{?}{\geq} -\log Z$$

An example game

MP



CP



$x_{t_1} x_{t_2} x_{t_3}$

1		
---	--	--

1	0	
---	---	--

1	0	1
---	---	---

$t_1 = 1$

$x_{t_1}^* = 1$

$Q(x_1^*) = 0.2$

$t_2 = 2$

$x_{t_2}^* = 0$

$Q(x_2^* | x_1^*) = 0.2$

$t_3 = 3$

$x_{t_3}^* = 1$

$Q(x_3^* | x_{1,2}^*) = 0.5$

$$x^* = (1, 0, 1), \quad Q(x^*) = 0.2 \times 0.2 \times 0.5 = 0.02$$

$$V^+(Q, R) = -\log 500 \stackrel{?}{\geq} -\log Z$$

Who wins?

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Theoretical results

Experimental results

Conclusion

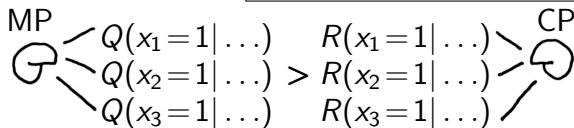
The difference score

MP: advertises marginals $Q(x_i|x_{1:i-1}^*)$
CP: chooses $x_i^* = \arg \max_{x_i} \frac{Q(x_i|x_{1:i-1}^*)}{R(x_i|x_{1:i-1}^*)}$

The difference score

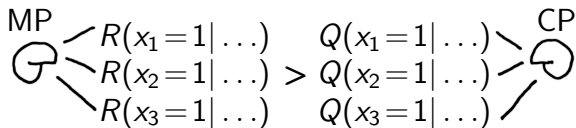
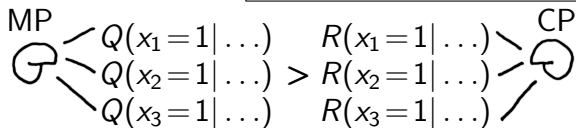
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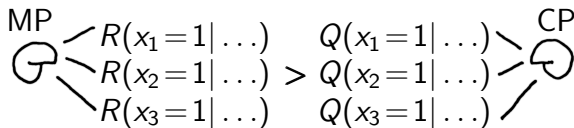
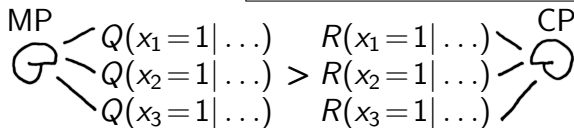
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The difference score

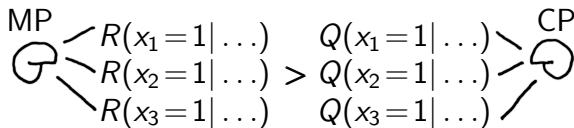
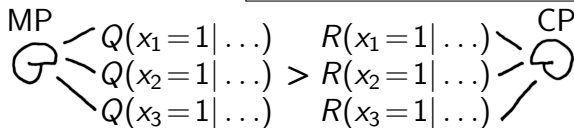
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$$S^+(Q, R) = V^+(\text{MP} = Q, \text{CP} = R) \\ - V^+(\text{MP} = R, \text{CP} = Q)$$

The difference score

MP: advertises marginals $Q(x_i|x_{1:i-1}^*)$
CP: chooses $x_i^* = \arg \max_{x_i} \frac{Q(x_i|x_{1:i-1}^*)}{R(x_i|x_{1:i-1}^*)}$



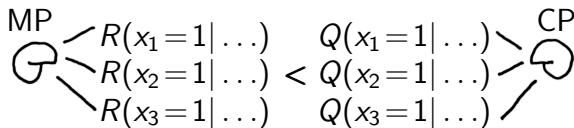
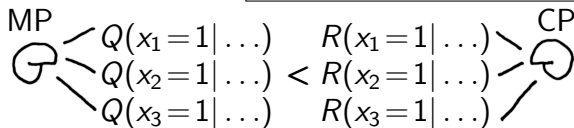
$$S^+(Q, R) = V^+(\text{MP} = Q, \text{CP} = R) \\ - V^+(\text{MP} = R, \text{CP} = Q)$$

Note that

$$S^+(P, Q) \leq 0$$

The difference score

MP: advertises marginals $Q(x_i|x_{1:i-1}^*)$
CP: chooses $x_i^* = \arg \max_{x_i} \frac{Q(x_i|x_{1:i-1}^*)}{R(x_i|x_{1:i-1}^*)}$



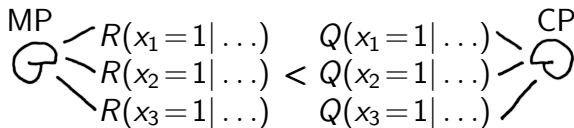
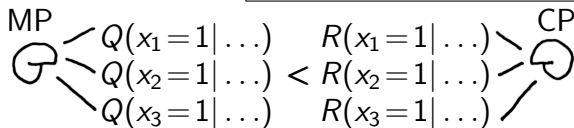
$$S^-(Q, R) = V^-(\text{MP} = Q, \text{CP} = R) \\ - V^-(\text{MP} = R, \text{CP} = Q)$$

Note that

$$S^-(P, Q) \geq 0$$

The difference score

MP: advertises marginals $Q(x_i|x_{1:i-1}^*)$
CP: chooses $x_i^* = \arg \max_{x_i} \frac{Q(x_i|x_{1:i-1}^*)}{R(x_i|x_{1:i-1}^*)}$



$$S^-(Q, R) = V^-(\text{MP} = Q, \text{CP} = R) \\ - V^-(\text{MP} = R, \text{CP} = Q)$$

Note that

$$S^-(P, Q) \geq 0$$

S^+ penalises overestimates, S^- penalises underestimates

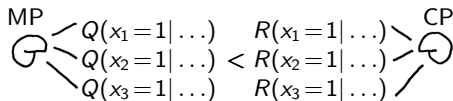
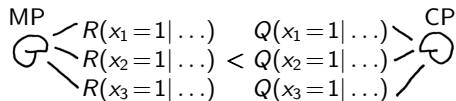
The four-way score

MP: advertises marginals $Q(x_i|x_{1:i-1}^*)$
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The four-way score

MP: advertises marginals $Q(x_i|x_{1:i-1}^*)$

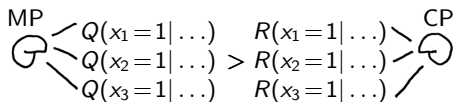
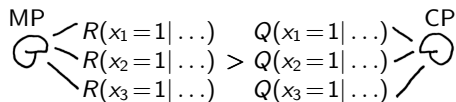
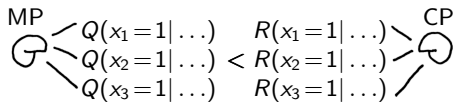
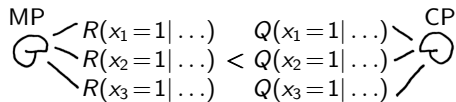
CP: chooses $x_i^* = \arg \max_{x_i} \frac{Q(x_i|x_{1:i-1}^*)}{R(x_i|x_{1:i-1}^*)}$



The four-way score

MP: advertises marginals $Q(x_i|x_{1:i-1}^*)$

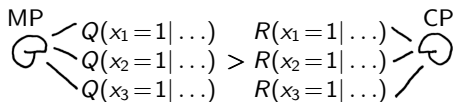
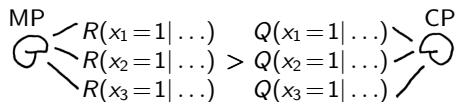
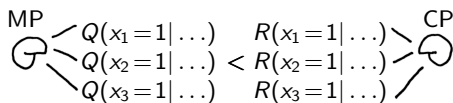
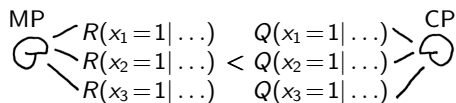
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The four-way score

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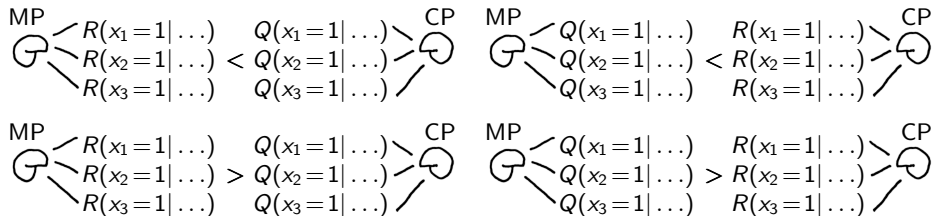


$$S_4(Q, R) = S^-(Q, R) - S^+(Q, R)$$

The four-way score

MP: advertises marginals $Q(x_i|x_{1:i-1}^*)$

CP: chooses $x_i^* = \arg \max_{x_i} \frac{Q(x_i|x_{1:i-1}^*)}{R(x_i|x_{1:i-1}^*)}$

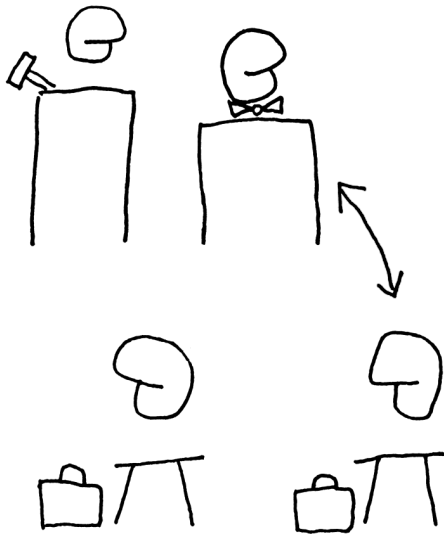


$$S_4(Q, R) = S^-(Q, R) - S^+(Q, R)$$

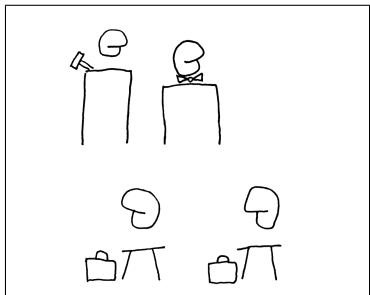
Note that

$$S_4(P, Q) \geq 0$$



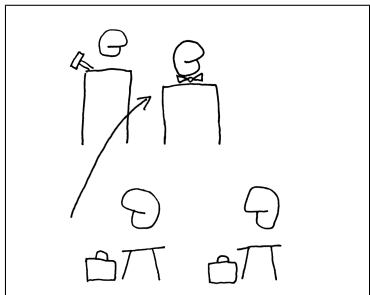


Legal analogy



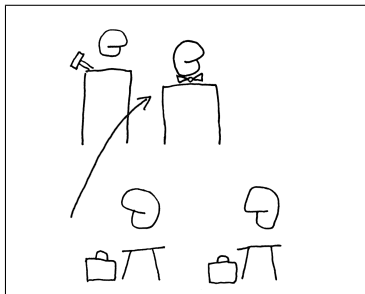
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Defense

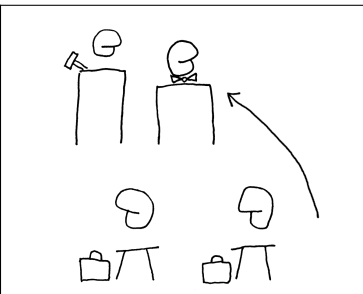


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Prosecution

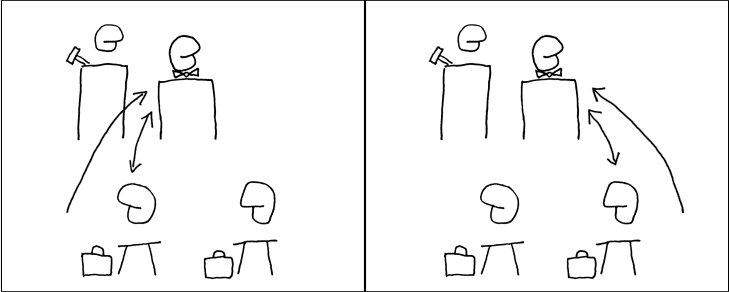


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Examined

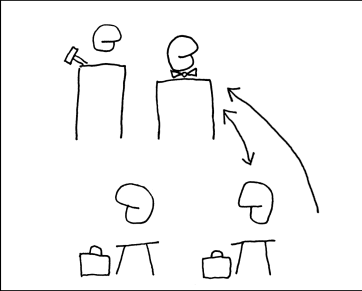
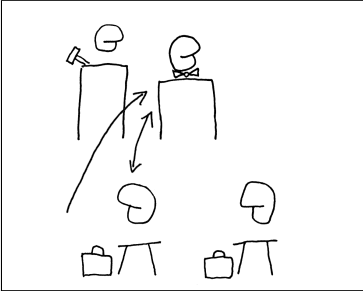


Legal analogy

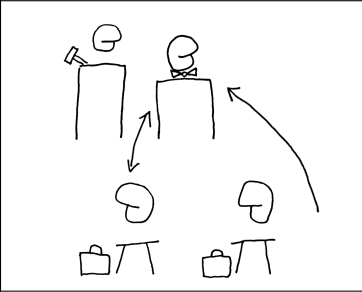
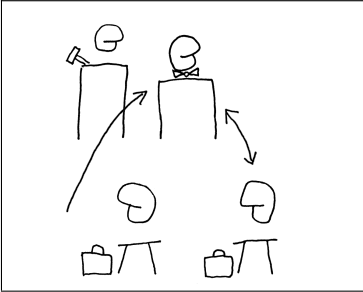
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Properties of four-way score

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What if neither Q nor R is exact?

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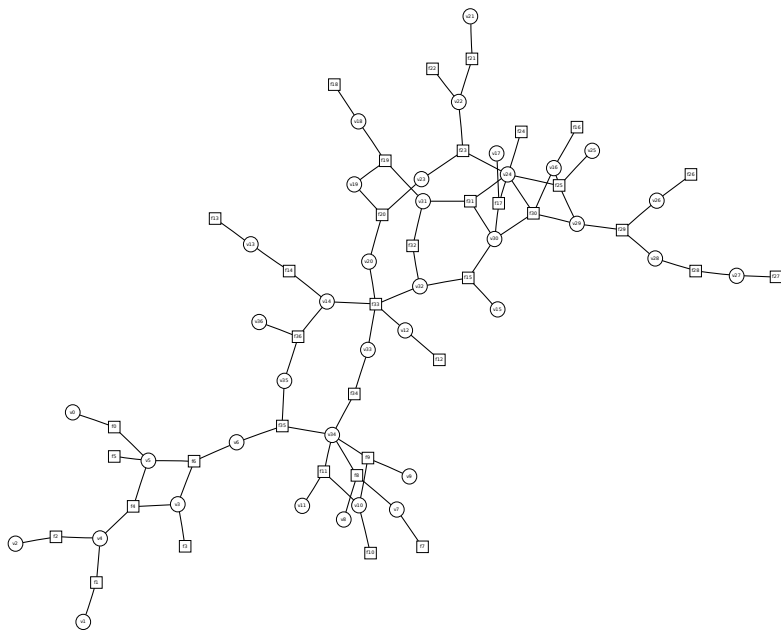
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Comparison of five approximations

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► Errors:

	avg L_1
LCBP	0.0001
TreeEP	0.0087
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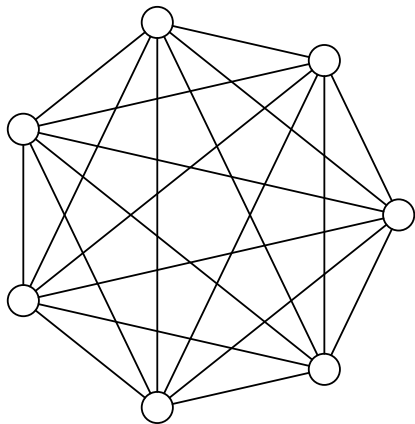
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► Pairwise S_4 scores:

	LCBP	TreeEP	CBP	BP	Gibbs
LCBP	0	5.3	13.8	22.8	13.0
TreeEP		0	8.4	13.5	4.0
CBP			0	27.6	3.7
BP				0	-4.0
Gibbs					0

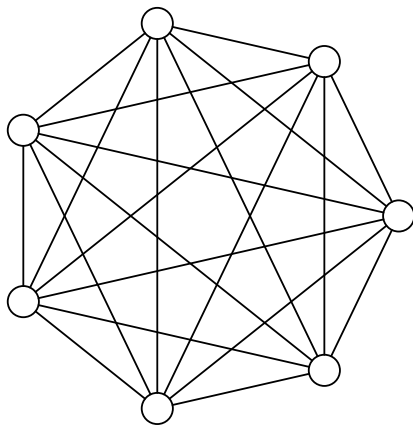
Random approximations and models

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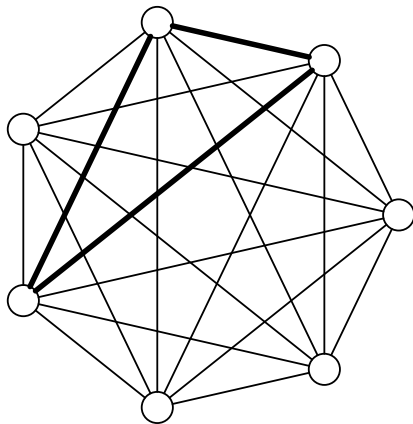
- ▶ Fully connected binary pairwise, 7 variables

Random approximations and models



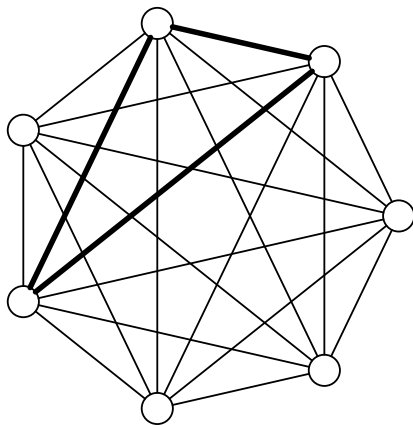
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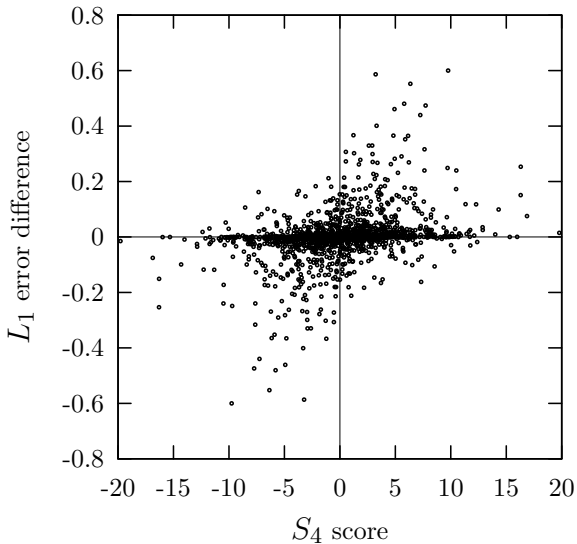
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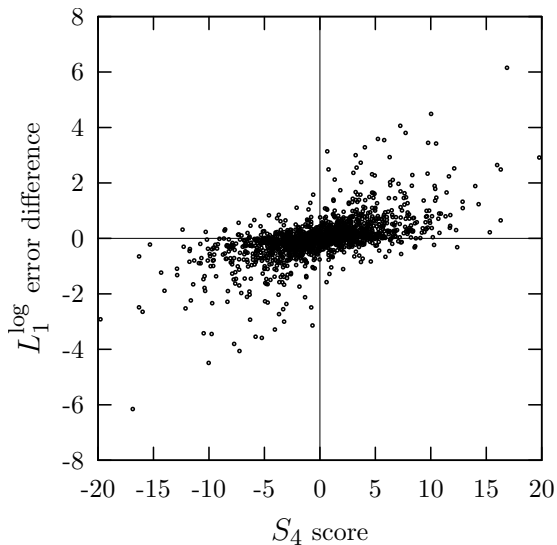
- ▶ Fully connected binary pairwise, 7 variables
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- ▶ (really HAK)

S_4 versus error



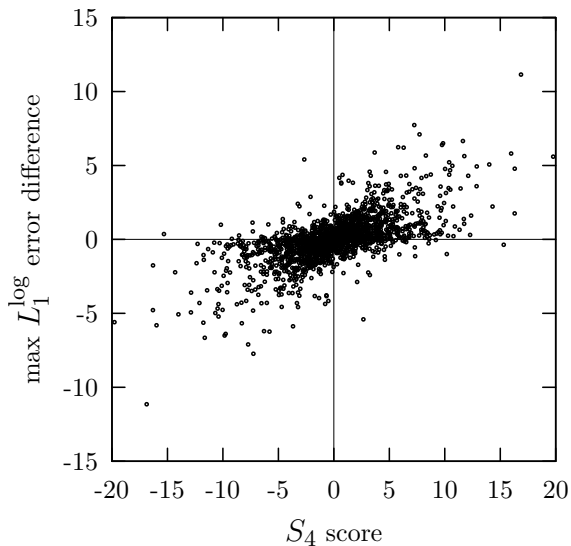
Average L_1 error: 64% agreement

S_4 versus error

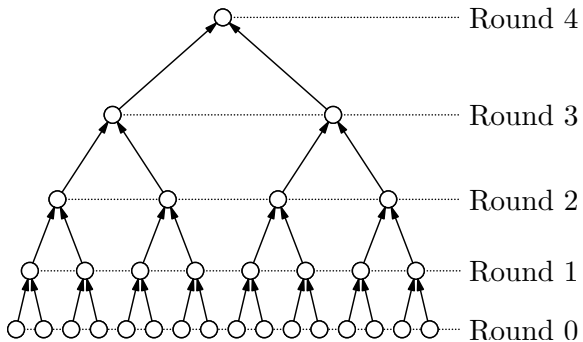


Average L_1^{\log} error: 75% agreement

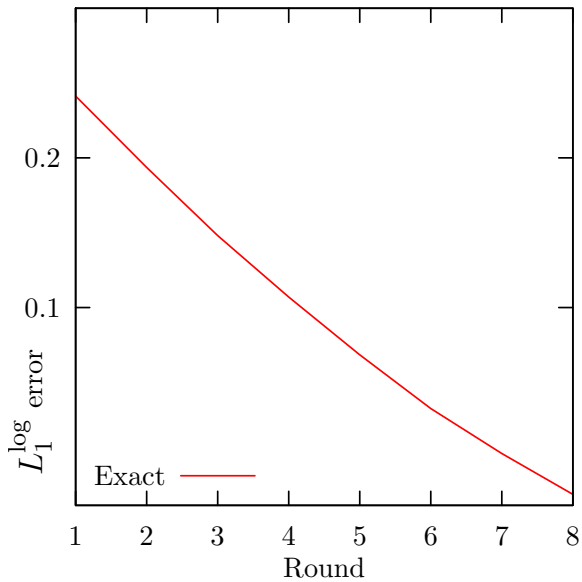
S_4 versus error



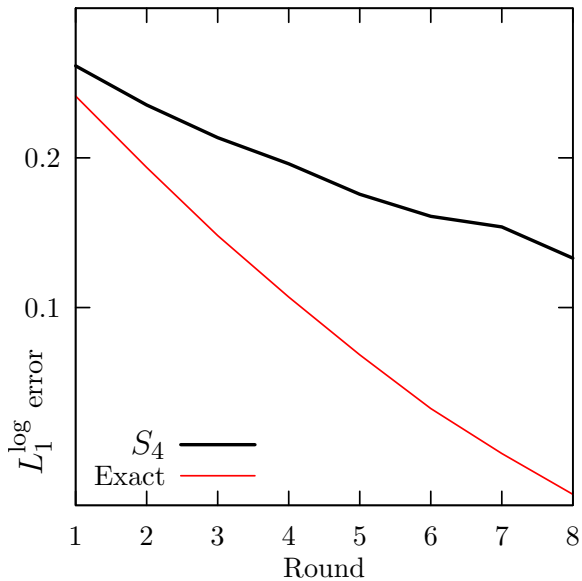
Maximum L_1^{\log} error: 77% agreement



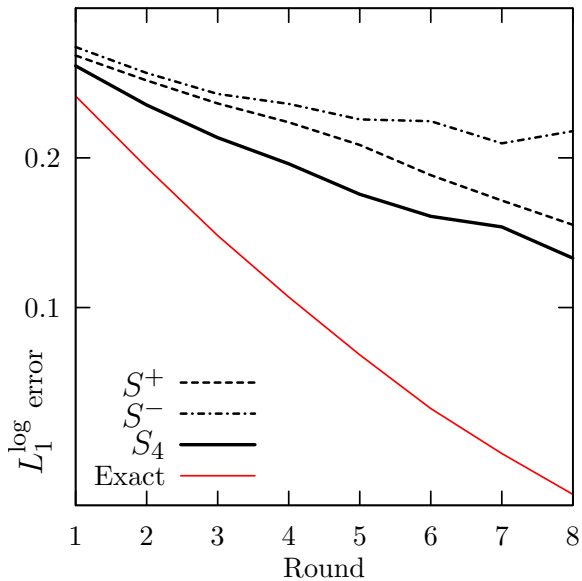
Single-elimination tournament



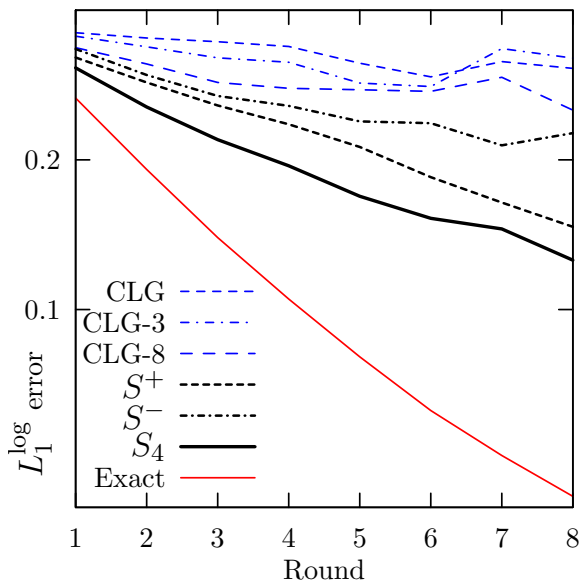
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(Topsøe 1979)

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- ▶ Approximations with compilation (e.g. (Lowd 2010))