Outline

A conditional game for comparing approximations

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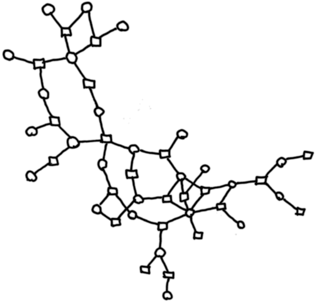
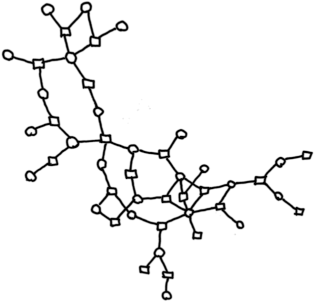
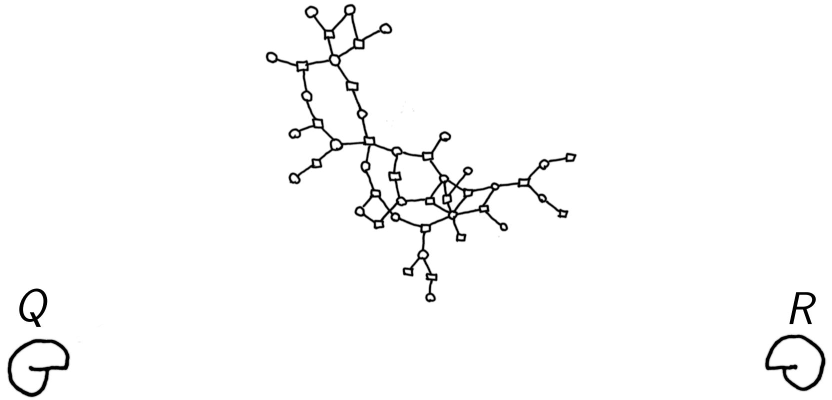
model $\rightarrow$

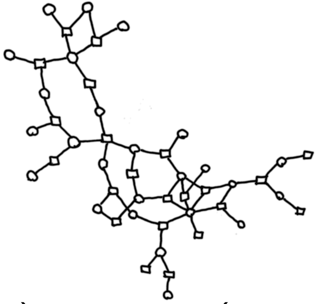


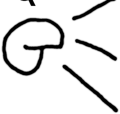
model →

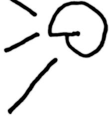


approximate inference algorithm →


Q
R







$$Q(x_1=1)=0.5 \quad R(x_1=1)=0.4$$
$$Q(x_2=1)=0.3 \quad R(x_2=1)=0.4$$
$$Q(x_3=1)=0.8 \quad R(x_3=1)=0.7$$

Q

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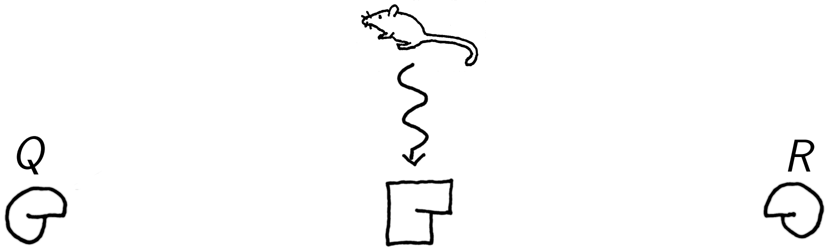
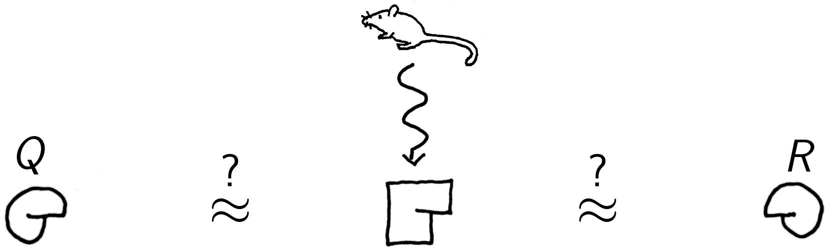
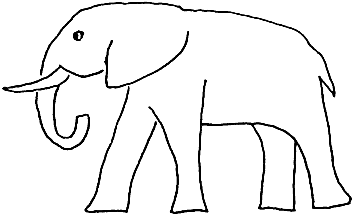
R


Q
R



Q
?
≈
?
≈
R



Q
?
≈
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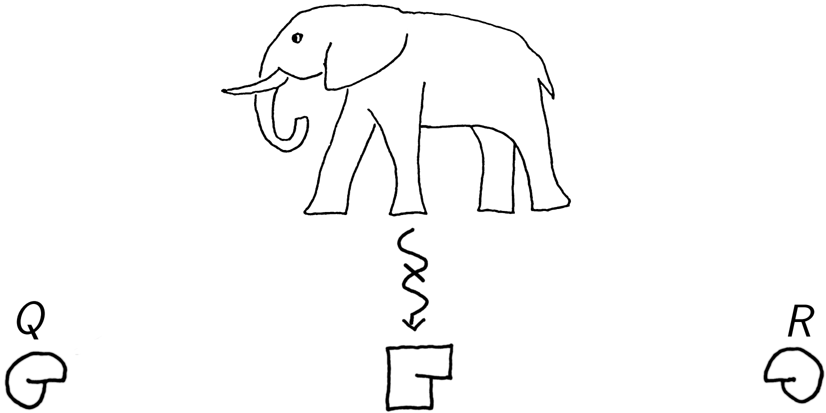
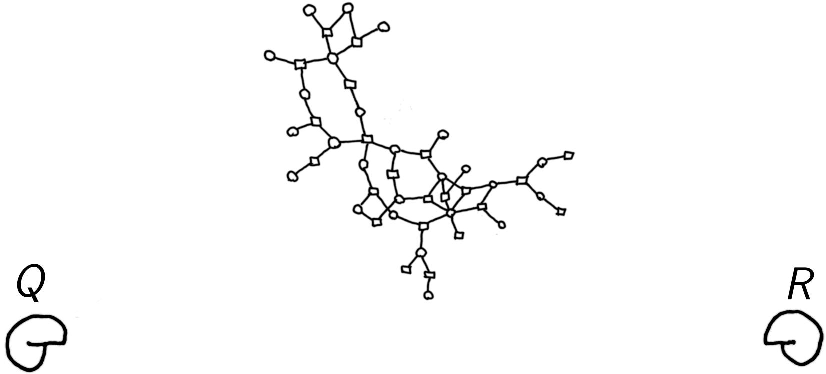
≈
R


Q
R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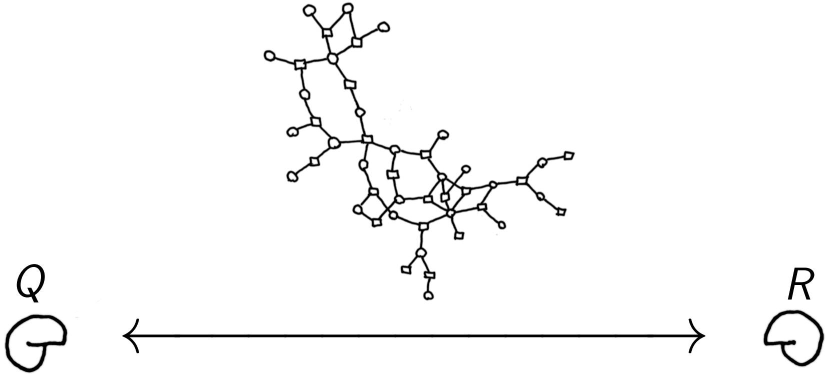
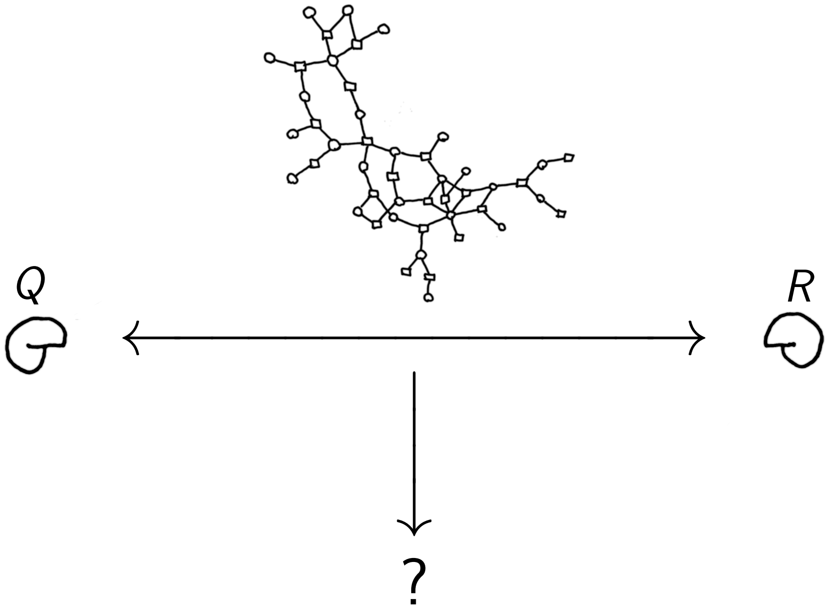
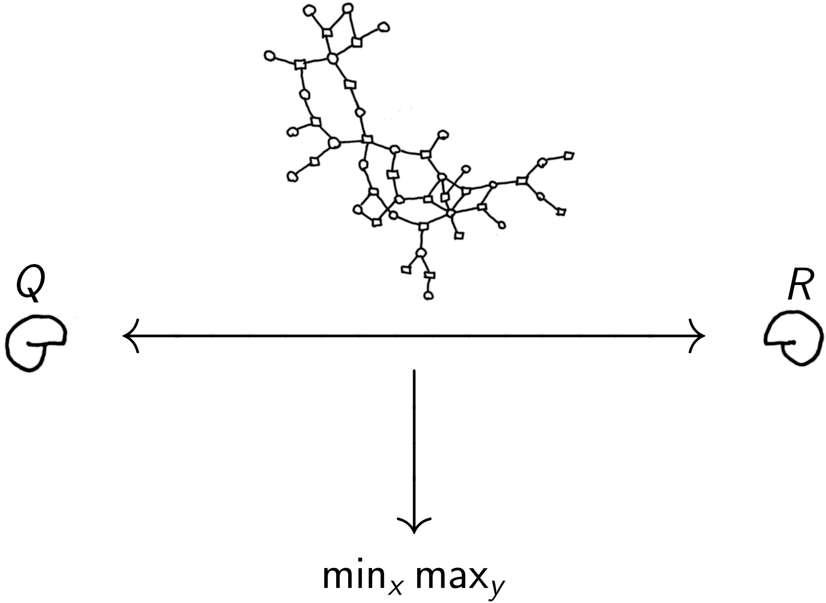
R

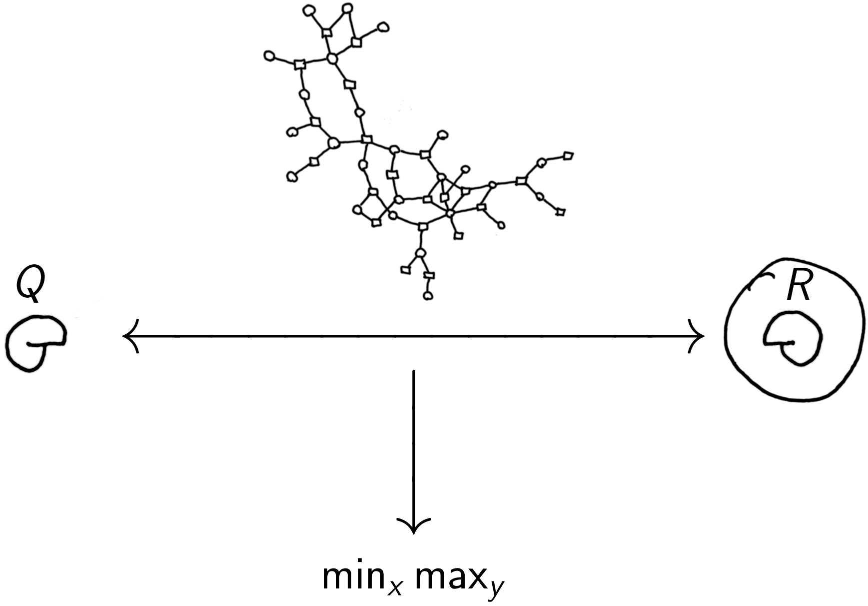


Q
R


Q
R
?



Q
R
$\min_x \max_y$


Q
R
min_x max_y

Overview

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- Exact inference is optimal
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- Empirical investigations
- Some pros, some cons. Future work

Outline

Approximate Inference

random state $x = (x_1, \ldots, x_i, \ldots, x_n)$

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$$\prod_{\alpha} \psi_\alpha(x_\alpha)$$

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$$P(x) = \frac{1}{Z} \prod_{\alpha} \psi_\alpha(x_\alpha)$$

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$$\sum_{x_{\setminus i}} P(x)$$

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$$\sum_{x_{\setminus i}} P(x) = P(x_i)$$

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$$P(x) = \frac{1}{Z} \prod_{\alpha} \psi_{\alpha}(x_{\alpha})$$

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$$Q(x_i) \approx \sum_{x_{\setminus i}} P(x) = P(x_i)$$

approximate inference

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exact inference

Approximate Inference

random state $x = (x_1, \ldots, x_i, \ldots, x_n)$, factors α

$$\delta(x_k, x_k^*) \prod_\alpha \psi_\alpha(x_\alpha)$$

Approximate Inference

random state $x = (x_1, \ldots, x_i, \ldots, x_n)$, factors α

$$P'(x) = \frac{1}{Z'} \delta(x_k, x_k^*) \prod_{\alpha} \psi_\alpha(x_\alpha)$$

Approximate Inference

random state $x = (x_1, \ldots, x_i, \ldots, x_n)$, factors α

$$P'(x) = \frac{1}{Z'}\delta(x_k, x_k^*)\prod_{\alpha}\psi_\alpha(x_\alpha)$$

$$\left(Z' = \sum_{x}\delta(x_k, x_k^*)\prod_{\alpha}\psi_\alpha(x_\alpha)\right)$$

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conditioned model

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$$Q(x_i) \approx \sum_{x_{\setminus i}} P'(x)$$

conditioned model

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$$Q(x_i) \approx \sum_{x_{\setminus i}} P'(x) = P(x_i | x_k = x_k^*)$$

conditioned model

Outline

Game requirements

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1. No extra computation

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$$ZP(x) = \prod_{\alpha} \psi_\alpha(x_\alpha)$$

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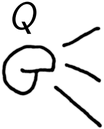
1. No extra computation

$$ZP(x) = \prod_{\alpha} \psi_\alpha(x_\alpha)$$

2. No additional randomness



$$Q(x_1 = 1) = 0.5$$
$$Q(x_2 = 1) = 0.3$$
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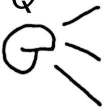
Q



$Q(x_1 = 1) = 0.5$

$Q(x_2 = 1) = 0.3$

$Q(x_3 = 1) = 0.8$

$\downarrow$?

$$(x_1, x_2, x_3) = (1, 0, 1)$$

Sampling a point

Sampling a point

x_1	x_2	x_3
?		

$$Q(x_1 = 1) = 0.5$$

Sampling a point

x_1	x_2	x_3
?		
1	?	

$$Q(x_1 = 1) = 0.5$$

$$Q(x_2 = 0 | x_1 = 1) = 0.2$$

Sampling a point

x_1	x_2	x_3
?		
1	?	
1	0	?

$$Q(x_1 = 1) = 0.5$$

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$$Q(x_3 = 1 | x_1 = 1, x_2 = 0) = 0.7$$

Sampling a point

x_1	x_2	x_3
?		
1	?	
1	0	?
1	0	1

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$$\longrightarrow ZP(x) = \prod_{\alpha} \psi_\alpha(x_1 = 1, x_2 = 0, x_3 = 1)$$

Sampling a point

x_1	x_2	x_3
?		
1	?	
1	0	?
1	0	1

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$$Q(x_2 = 0 | x_1 = 1) = 0.2$$

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$$\longrightarrow Q(x) = Q(x_1 = 1) \times Q(x_2 = 0 | x_1 = 1) \times Q(x_3 = 1 | x_1 = 1, x_2 = 0)$$

Two quantities

$$
\begin{aligned}
Q(x) &= \prod_i Q(x_i | x_{1:i-1}) \\
ZP(x) &= \prod_\alpha \psi_\alpha(x_\alpha)
\end{aligned}
$$

Two quantities

$$\frac{Q(x) \;=\; \prod_i Q(x_i|x_{1:i-1})}{ZP(x) \;=\; \prod_\alpha \psi_\alpha(x_\alpha)}$$

Two quantities

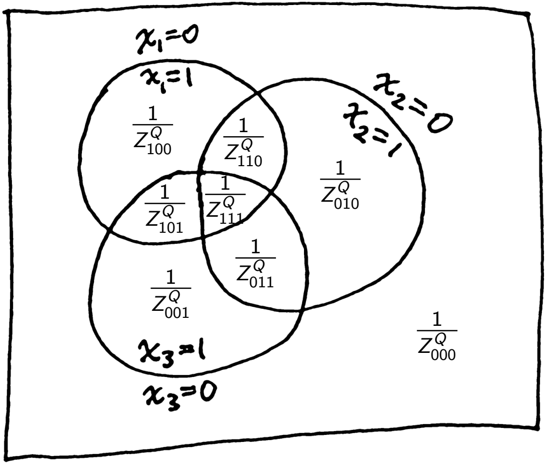
$$\frac{\begin{array}{rcl} Q(x) & = & \prod_i Q(x_i | x_{1:i-1}) \end{array}}{\begin{array}{rcl} ZP(x) & = & \prod_\alpha \psi_\alpha(x_\alpha) \end{array}} \approx \frac{1}{Z}$$

Two quantities

$$\frac{\begin{array}{rcl} Q(x) & = & \prod_i Q(x_i|x_{1:i-1}) \end{array}}{\begin{array}{rcl} ZP(x) & = & \prod_\alpha \psi_\alpha(x_\alpha) \end{array}} = \frac{1}{Z_x^Q}$$

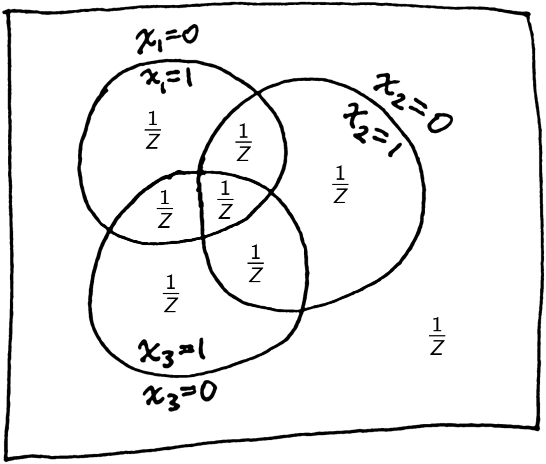
Partition function estimate

$$\frac{1}{Z_x^Q} = \frac{Q(x)}{ZP(x)}$$



Partition function estimate

$$\frac{1}{Z_x^Q} = \frac{P(x)}{ZP(x)} = \frac{1}{Z}$$



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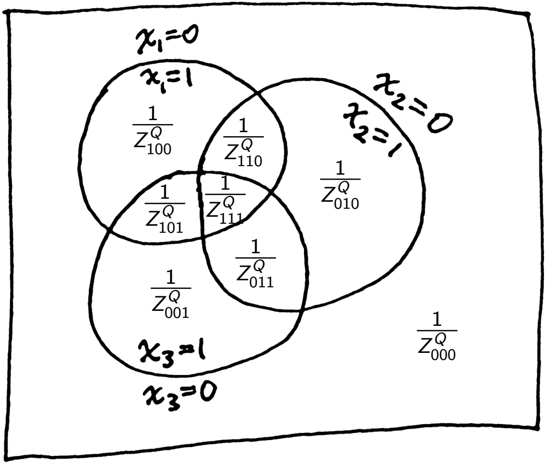
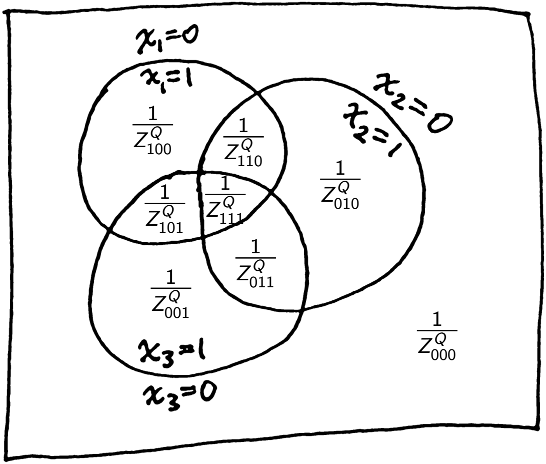


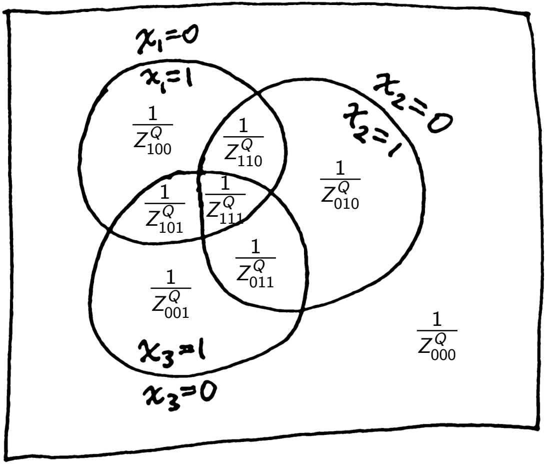
Partition function estimate

$$\frac{1}{Z_x^Q} Z P(x) = Q(x)$$



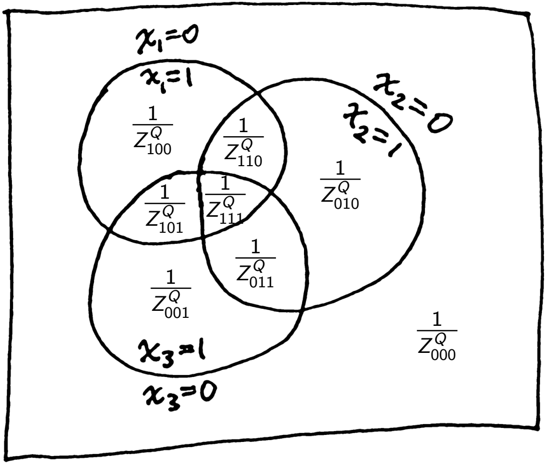
Partition function estimate

$$\sum_x \left(\frac{1}{Z_x^Q} ZP(x) = Q(x) \right) = 1$$



Partition function estimate

$$\sum_x \left(\frac{1}{Z_x^Q} \underline{ZP(x)} = Q(x) \right) = 1$$



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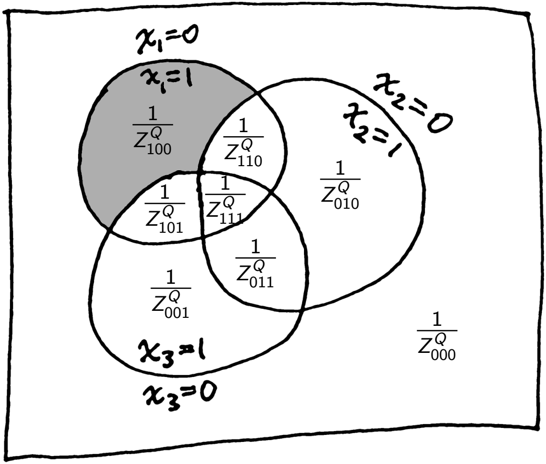

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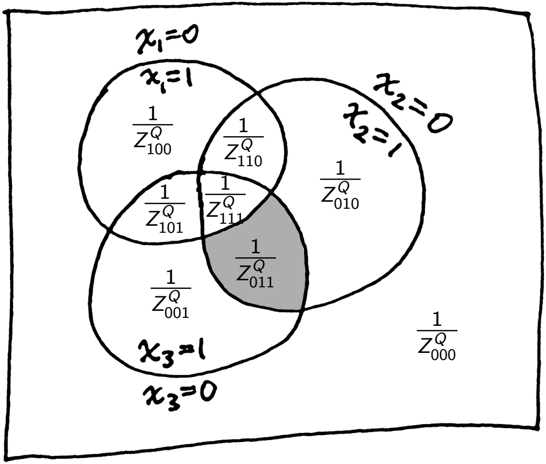

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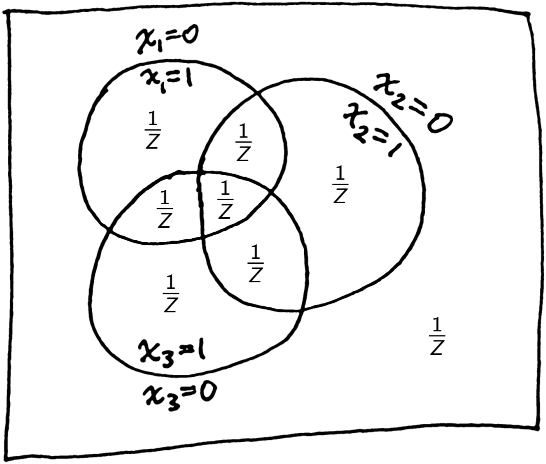
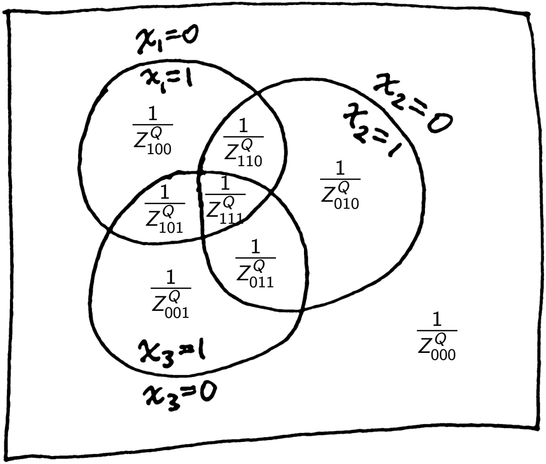


The opponent's strategy

$$Q(x_1) = \sum_{x_{\setminus 1}} \left(Q(x) = \frac{1}{Z_x^Q} Z P(x) \right)$$



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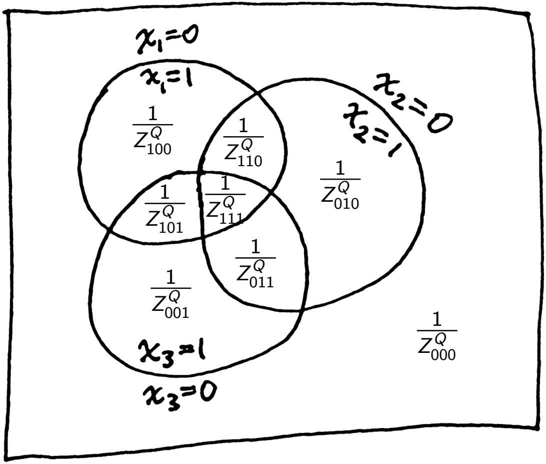
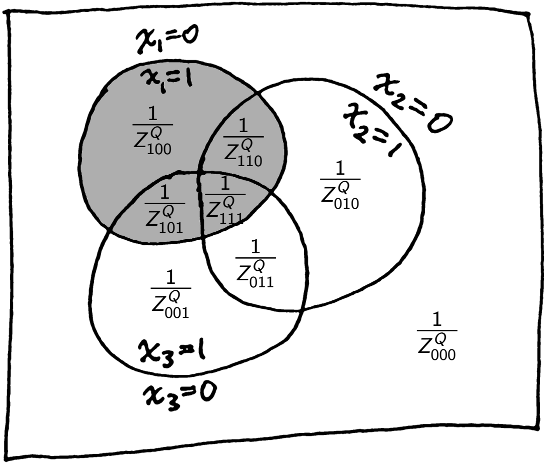


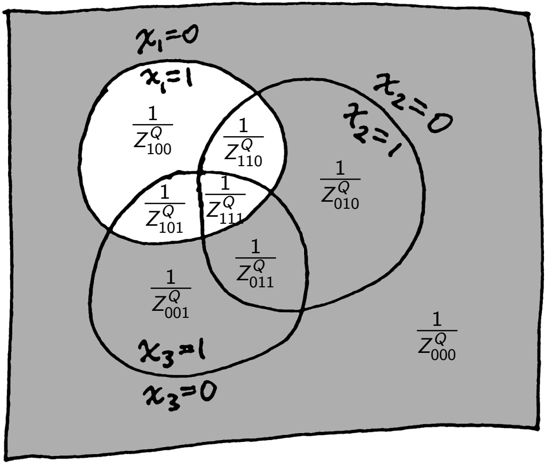
The opponent's strategy

$$P(x_1 = 1) < Q(x_1 = 1) = \sum_{x_{\setminus 1}} \left(Q(x) = \frac{1}{Z_x^Q} \underline{ZP(x)} \right)$$



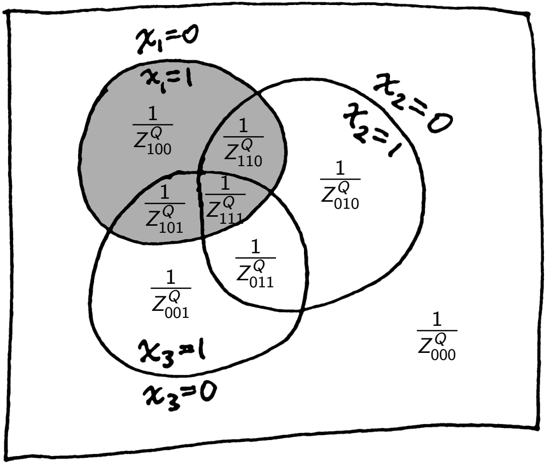
The opponent's strategy

$$P(x_1 = 0) < Q(x_1 = 0) = \sum_{x_{\setminus 1}} \left(Q(x) = \frac{1}{Z_x^Q} \underline{ZP(x)} \right)$$



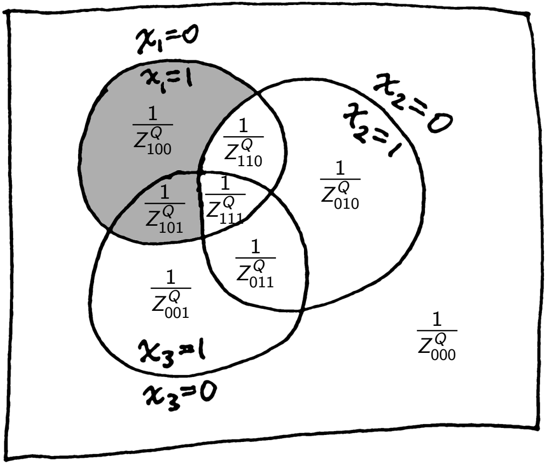
The opponent's strategy

$$R(x_1) \gtrless \sum_{x_{\setminus 1}} \left(Q(x) = \frac{1}{Z_x^Q} \underline{ZP(x)} \right)$$



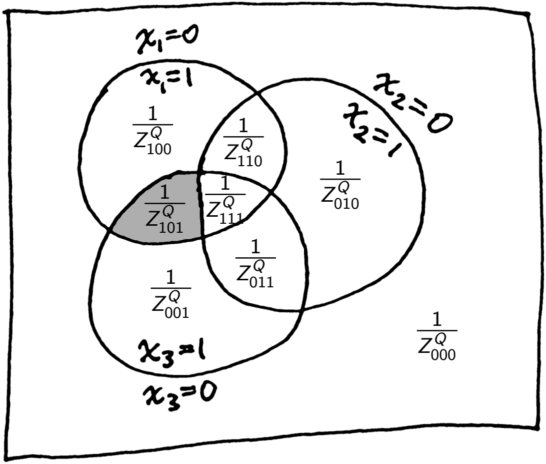
The opponent's strategy

$$R(x_2, x_1) \gtrless \sum_{x_{\setminus 1,2}} \left(Q(x) = \frac{1}{Z_x^Q} \underline{ZP(x)} \right)$$



The opponent's strategy

$$R(x_3, x_1, x_2) \gtrless Q(x) = \frac{1}{Z^Q_x} \underline{ZP(x)}$$



Strategies versus conditioned marginals

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- In general, strategies $\supset$ approximations
- Possible to simulate opponent
- Unfold game tree for several moves
- $\implies$ Too expensive
- Better to pretend opponent unknown, second-guessing impossible
- Assume players use natural strategies

The opponent's strategy, restated

$$\frac{Q(x)}{ZP(x)} = \frac{1}{Z_x^Q}$$

The opponent's strategy, restated

$$\frac{Q(x)}{ZP(x)} = \frac{1}{Z_x^Q}$$

$$\frac{\begin{array}{r} Q(x_1) \\ \times Q(x_2|x_1) \\ \times Q(x_3|x_1, x_2) \end{array}}{ZP(x)}$$

The opponent's strategy, restated

$$\frac{Q(x)}{ZP(x)} = \frac{1}{Z_x^Q}$$

$$\frac{\begin{array}{r} Q(x_1) \\ \times Q(x_2|x_1) \\ \times Q(x_3|x_1,x_2) \end{array}}{ZP(x)} \quad \geq \quad R(x_1)$$

The opponent's strategy, restated

$$\frac{Q(x)}{ZP(x)} = \frac{1}{Z_x^Q}$$

$$\begin{array}{rcl} Q(x_1) & \geq & R(x_1) \\ \times Q(x_2|x_1) & \geq & R(x_2|x_1) \\ \times Q(x_3|x_1, x_2) & & \\ \hline ZP(x) & & \end{array}$$

The opponent's strategy, restated

$$\frac{Q(x)}{ZP(x)} = \frac{1}{Z_x^Q}$$

$$\frac{\begin{array}{r} Q(x_1) \\ \times Q(x_2|x_1) \\ \times Q(x_3|x_1,x_2) \end{array}}{ZP(x)} \quad \begin{array}{ccl} \geq & R(x_1) \\ \geq & R(x_2|x_1) \\ \geq & R(x_3|x_1,x_2) \end{array}$$

The opponent's strategy, restated

$$\frac{Q(x)}{ZP(x)} = \frac{1}{Z_x^Q} \geq \frac{1}{Z_x^R}$$

$$\begin{array}{rcl} Q(x_1) & \geq & R(x_1) \\ \times Q(x_2|x_1) & \geq & R(x_2|x_1) \\ \times Q(x_3|x_1, x_2) & \geq & R(x_3|x_1, x_2) \\ \hline ZP(x) & & \end{array}$$

Outline

Definition of conditional game

Marginal player

Definition of conditional game

Marginal player, conditional player

Definition of conditional game

Marginal player, conditional player (Q, R)

Definition of conditional game

Marginal player, conditional player (Q, R)

Turn $i \in 1 \ldots n$:

Definition of conditional game

Marginal player, conditional player (Q, R)
Turn $i \in 1 \ldots n$:

- MP: advertises marginals $Q(x_i | x^*_{1:i-1})$

Definition of conditional game

Marginal player, conditional player (Q, R)
Turn $i \in 1 \ldots n$:

- MP: advertises marginals $Q(x_i | x^*_{1:i-1})$
- CP: chooses a state x^*_i

Definition of conditional game

Marginal player, conditional player (Q, R)
Turn $i \in 1 \ldots n$:

- MP: advertises marginals $Q(x_i | x^*_{1:i-1})$
- CP: chooses a state x^*_i

$$x^*_i = \arg\max_{x_i} \frac{Q(x_i | x^*_{1:i-1})}{R(x_i | x^*_{1:i-1})}$$

Definition of conditional game

Marginal player, conditional player (Q, R)
Turn $i \in 1 \ldots n$:

- MP: advertises marginals $Q(x_i | x^*_{1:i-1})$
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$$x^*_i = \arg\max_{x_i} \frac{Q(x_i | x^*_{1:i-1})}{R(x_i | x^*_{1:i-1})}$$

$\left(\rightarrow x^*, \{Q(x^*_i | x^*_{1:i-1})\}_i\right)$

Definition of conditional game

Marginal player, conditional player $\quad (Q, R)$
Turn $i \in 1 \ldots n$:

- MP: advertises marginals $Q(x_i|x^*_{1:i-1})$
- CP: chooses a state x^*_i

$$x^*_i = \arg\max_{x_i} \frac{Q(x_i|x^*_{1:i-1})}{R(x_i|x^*_{1:i-1})}$$

$\left(\to x^*, \{Q(x^*_i|x^*_{1:i-1})\}_i\right)$
Value of game:

$$V^+(Q, R) \equiv \log \frac{1}{Z^Q_{x^*}} = \log \frac{\prod_i Q(x^*_i|x^*_{1:i-1})}{\prod_\alpha \psi_\alpha(x^*_\alpha)}$$

Definition of conditional game

Marginal player, conditional player (Q, R)
Turn $i \in 1 \ldots n$:

- MP: advertises marginals $Q(x_i | x^*_{1:i-1})$
- CP: chooses a state x^*_i

$$x^*_i = \arg\max_{x_i} \frac{Q(x_i | x^*_{1:i-1})}{R(x_i | x^*_{1:i-1})}$$

$(\rightarrow x^*, \{Q(x^*_i | x^*_{1:i-1})\}_i)$
Value of game:

$$V^+(Q, R) \equiv \log \frac{1}{Z^Q_{x^*}} = \log \frac{\prod_i Q(x^*_i | x^*_{1:i-1})}{\prod_\alpha \psi_\alpha(x^*_\alpha)}$$

Definition of conditional game

Marginal player, conditional player (Q, R)
Turn $i \in 1 \ldots n$:

- MP: advertises marginals $Q(x_i|x^*_{1:i-1})$
- CP: chooses a state x^*_i

$$x^*_i = \operatorname*{arg\,min}_{x_i} \frac{Q(x_i|x^*_{1:i-1})}{R(x_i|x^*_{1:i-1})}$$

$(\rightarrow x^*, \{Q(x^*_i|x^*_{1:i-1})\}_i)$
Value of game:

$$V^-(Q,R) \equiv \log \frac{1}{Z^Q_{x^*}} = \log \frac{\prod_i Q(x^*_i|x^*_{1:i-1})}{\prod_\alpha \psi_\alpha(x^*_\alpha)}$$

Variable order

Turn $i \in 1 \ldots n$:

Value:	CP chooses:
$V^+(Q, R)$	$\arg\max_{x_i} \frac{Q(x_i \mid x^*_{1:i-1})}{R(x_i \mid x^*_{1:i-1})}$
$V^-(Q, R)$	$\arg\min_{x_i} \frac{Q(x_i \mid x^*_{1:i-1})}{R(x_i \mid x^*_{1:i-1})}$

$$V^+ = V^- = \log \frac{1}{Z_x^Q}$$

Variable order

Turn $t \in 1 \ldots n$:

Value:	CP chooses:
$V^+(Q, R)$	$\arg\max\limits_{(i_t \notin i_{1:t-1}, x_{i_t})} \frac{Q(x_{i_t} \mid x^*_{i_{1:t-1}})}{R(x_{i_t} \mid x^*_{i_{1:t-1}})}$
$V^-(Q, R)$	$\arg\min\limits_{(i_t \notin i_{1:t-1}, x_{i_t})} \frac{Q(x_{i_t} \mid x^*_{i_{1:t-1}})}{R(x_{i_t} \mid x^*_{i_{1:t-1}})}$

$$V^+ = V^- = \log \frac{1}{Z_x^Q}$$

An example game

MP

$Q(x_1 = 1) = 0.2$
$Q(x_2 = 1) = 0.7$
$Q(x_3 = 1) = 0.4$

$R(x_1 = 1) = 0.1$
$R(x_2 = 1) = 0.8$
$R(x_3 = 1) = 0.3$

CP

An example game

MP

$Q(x_1 = 1) = 0.2$
$Q(x_2 = 1) = 0.7$
$Q(x_3 = 1) = 0.4$

$R(x_1 = 1) = 0.1$
$R(x_2 = 1) = 0.8$
$R(x_3 = 1) = 0.3$

CP

x_{t_1}	x_{t_2}	x_{t_3}
1		

$t_1 = 1$ $\quad x^*_{t_1} = 1$ $\quad Q(x^*_1) = 0.2$

An example game

MP

$$Q(x_1=1|x_1=1)=1$$
$$Q(x_2=1|x_1=1)=0.8$$
$$Q(x_3=1|x_1=1)=0.3$$

CP

$$R(x_1=1|\ldots)=1$$
$$R(x_2=1|\ldots)=0.9$$
$$R(x_3=1|\ldots)=0.2$$

x_{t_1}	x_{t_2}	x_{t_3}
1		

$t_1 = 1$ $\quad x^*_{t_1} = 1$ $\quad Q(x^*_1) = 0.2$

An example game

MP

$$Q(x_1 = 1 | x_1 = 1) = 1$$
$$Q(x_2 = 1 | x_1 = 1) = 0.8$$
$$Q(x_3 = 1 | x_1 = 1) = 0.3$$

$$R(x_1 = 1 | \ldots) = 1$$
$$R(x_2 = 1 | \ldots) = 0.9$$
$$R(x_3 = 1 | \ldots) = 0.2$$

CP

x_{t_1}	x_{t_2}	x_{t_3}
1		
1	0	

$$t_1 = 1 \qquad x^*_{t_1} = 1 \qquad Q(x^*_1) = 0.2$$
$$t_2 = 2 \qquad x^*_{t_2} = 0 \qquad Q(x^*_2 | x^*_1) = 0.2$$

An example game

MP

$Q(x_1 = 1 | x_{1,2} = (1,0)) = 1$

$Q(x_2 = 1 | x_{1,2} = (1,0)) = 0$

$Q(x_3 = 1 | x_{1,2} = (1,0)) = 0.5$

$R(x_1 = 1 | \ldots) = 1$

$R(x_2 = 1 | \ldots) = 0$

$R(x_3 = 1 | \ldots) = 0.4$

CP

x_{t_1}	x_{t_2}	x_{t_3}
1		

1	0	

$t_1 = 1 \qquad x^*_{t_1} = 1 \qquad Q(x^*_1) = 0.2$

$t_2 = 2 \qquad x^*_{t_2} = 0 \qquad Q(x^*_2 | x^*_1) = 0.2$

An example game

MP

$$Q(x_1=1|x_{1,2}=(1,0))=1$$
$$Q(x_2=1|x_{1,2}=(1,0))=0$$
$$Q(x_3=1|x_{1,2}=(1,0))=0.5$$

$$R(x_1=1|\dots)=1$$
$$R(x_2=1|\dots)=0$$
$$R(x_3=1|\dots)=0.4$$

CP

x_{t_1}	x_{t_2}	x_{t_3}
1		
1	0	
1	0	1

$$t_1 = 1 \qquad x^*_{t_1} = 1 \qquad Q(x^*_1) = 0.2$$
$$t_2 = 2 \qquad x^*_{t_2} = 0 \qquad Q(x^*_2|x^*_1) = 0.2$$
$$t_3 = 3 \qquad x^*_{t_3} = 1 \qquad Q(x^*_3|x^*_{1,2}) = 0.5$$

An example game

MP

CP

x_{t_1}	x_{t_2}	x_{t_3}
1		
1	0	
1	0	1

$$t_1 = 1 \qquad x^*_{t_1} = 1 \qquad Q(x^*_1) = 0.2$$

$$t_2 = 2 \qquad x^*_{t_2} = 0 \qquad Q(x^*_2 | x^*_1) = 0.2$$

$$t_3 = 3 \qquad x^*_{t_3} = 1 \qquad Q(x^*_3 | x^*_{1,2}) = 0.5$$

$$x^* = (1, 0, 1), \qquad Q(x^*) = 0.2 \times 0.2 \times 0.5 = 0.02$$

An example game

MP

CP

x_{t_1}	x_{t_2}	x_{t_3}
1		
1	0	
1	0	1

$$t_1 = 1 \qquad x^*_{t_1} = 1 \qquad Q(x^*_1) = 0.2$$

$$t_2 = 2 \qquad x^*_{t_2} = 0 \qquad Q(x^*_2 | x^*_1) = 0.2$$

$$t_3 = 3 \qquad x^*_{t_3} = 1 \qquad Q(x^*_3 | x^*_{1,2}) = 0.5$$

$$x^* = (1, 0, 1), \qquad Q(x^*) = 0.2 \times 0.2 \times 0.5 = 0.02$$

$$V^+(Q, R) = \log \frac{1}{Z^Q_{(1,0,1)}}$$

An example game

MP

CP

$x_{t_1} x_{t_2} x_{t_3}$

1		

1	0	

1	0	1

$$t_1 = 1 \qquad x^*_{t_1} = 1 \qquad Q(x^*_1) = 0.2$$

$$t_2 = 2 \qquad x^*_{t_2} = 0 \qquad Q(x^*_2 | x^*_1) = 0.2$$

$$t_3 = 3 \qquad x^*_{t_3} = 1 \qquad Q(x^*_3 | x^*_{1,2}) = 0.5$$

$$x^* = (1, 0, 1), \qquad Q(x^*) = 0.2 \times 0.2 \times 0.5 = 0.02$$

$$V^+(Q, R) = \log \frac{1}{Z^Q_{(1,0,1)}} = \log \frac{0.2 \times 0.2 \times 0.5}{\prod_\alpha \psi_\alpha(x^*_\alpha = (1,0,1))}$$

An example game

MP

CP

$x_{t_1} x_{t_2} x_{t_3}$

1		

1	0	

1	0	1

$$t_1 = 1 \qquad x^*_{t_1} = 1 \qquad Q(x^*_1) = 0.2$$

$$t_2 = 2 \qquad x^*_{t_2} = 0 \qquad Q(x^*_2 | x^*_1) = 0.2$$

$$t_3 = 3 \qquad x^*_{t_3} = 1 \qquad Q(x^*_3 | x^*_{1,2}) = 0.5$$

$$x^* = (1, 0, 1), \qquad Q(x^*) = 0.2 \times 0.2 \times 0.5 = 0.02$$

$$V^+(Q, R) = \log \frac{1}{Z^Q_{(1,0,1)}} = \log \frac{0.2 \times 0.2 \times 0.5}{\prod_\alpha \psi_\alpha(x^*_\alpha = (1,0,1))} = \log \frac{0.02}{10}$$

An example game

MP

CP

x_{t_1}	x_{t_2}	x_{t_3}
1		
1	0	
1	0	1

$$t_1 = 1 \qquad x^*_{t_1} = 1 \qquad Q(x^*_1) = 0.2$$

$$t_2 = 2 \qquad x^*_{t_2} = 0 \qquad Q(x^*_2 | x^*_1) = 0.2$$

$$t_3 = 3 \qquad x^*_{t_3} = 1 \qquad Q(x^*_3 | x^*_{1,2}) = 0.5$$

$$x^* = (1, 0, 1), \qquad Q(x^*) = 0.2 \times 0.2 \times 0.5 = 0.02$$

$$V^+(Q, R) = -\log 500$$

An example game

MP

CP

$x_{t_1} x_{t_2} x_{t_3}$

1		
1	0	
1	0	1

$$t_1 = 1 \qquad x^*_{t_1} = 1 \qquad Q(x^*_1) = 0.2$$

$$t_2 = 2 \qquad x^*_{t_2} = 0 \qquad Q(x^*_2 | x^*_1) = 0.2$$

$$t_3 = 3 \qquad x^*_{t_3} = 1 \qquad Q(x^*_3 | x^*_{1,2}) = 0.5$$

$$x^* = (1, 0, 1), \qquad Q(x^*) = 0.2 \times 0.2 \times 0.5 = 0.02$$

$$V^+(Q, R) = -\log 500 \overset{?}{\gtrless} -\log Z$$

An example game

MP 

CP 

x_{t_1}	x_{t_2}	x_{t_3}
1		

1	0	

1	0	1

$$t_1 = 1 \qquad x^*_{t_1} = 1 \qquad Q(x^*_1) = 0.2$$

$$t_2 = 2 \qquad x^*_{t_2} = 0 \qquad Q(x^*_2 | x^*_1) = 0.2$$

$$t_3 = 3 \qquad x^*_{t_3} = 1 \qquad Q(x^*_3 | x^*_{1,2}) = 0.5$$

$$x^* = (1, 0, 1), \qquad Q(x^*) = 0.2 \times 0.2 \times 0.5 = 0.02$$

$$V^+(Q, R) = -\log 500 \overset{?}{\gtrless} -\log Z$$

Who wins?

Outline

- Introduction
- Approximate inference
- Derivation of game
- Definition of game
- Scores for comparing approximations
- Theoretical results
- Experimental results
- Conclusion

The difference score

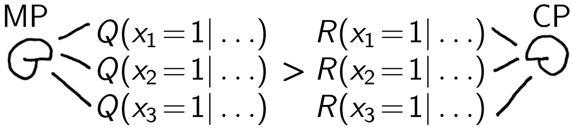
MP: advertises marginals $Q(x_i|x^*_{1:i-1})$

CP: chooses $x^*_i = \arg\left\{\begin{smallmatrix}\min\\ \max\end{smallmatrix}\right\}_{x_i} \frac{Q(x_i|x^*_{1:i-1})}{R(x_i|x^*_{1:i-1})}$

The difference score

MP: advertises marginals $Q(x_i|x^*_{1:i-1})$

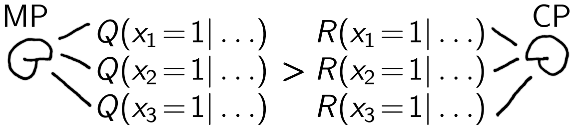
CP: chooses $x^*_i = \arg\left\{\begin{smallmatrix}\min\\\max\end{smallmatrix}\right\}_{x_i} \frac{Q(x_i|x^*_{1:i-1})}{R(x_i|x^*_{1:i-1})}$

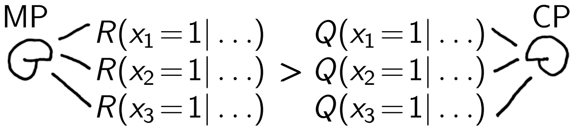


The difference score

MP: advertises marginals $Q(x_i|x^*_{1:i-1})$

CP: chooses $x^*_i = \arg\left\{\begin{matrix}\min\\ \max\end{matrix}\right\}_{x_i} \frac{Q(x_i|x^*_{1:i-1})}{R(x_i|x^*_{1:i-1})}$

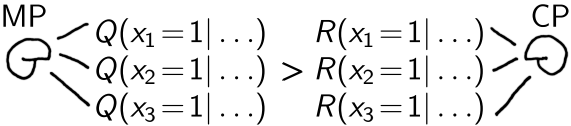


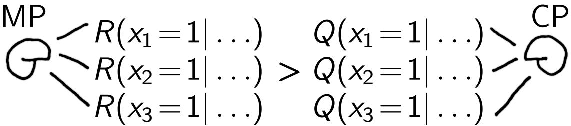


The difference score

MP: advertises marginals $Q(x_i|x^*_{1:i-1})$

CP: chooses $x^*_i = \arg\left\{ \begin{smallmatrix}\min\\ \max\end{smallmatrix} \right\}_{x_i} \frac{Q(x_i|x^*_{1:i-1})}{R(x_i|x^*_{1:i-1})}$



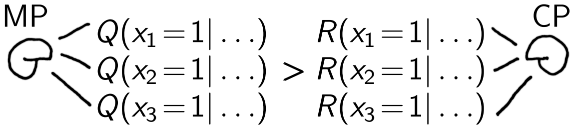


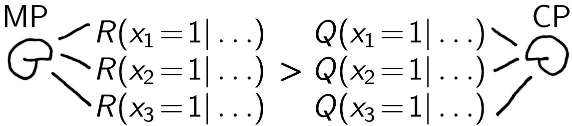
$$
\begin{aligned}
S^+(Q,R) &= V^+(\mathrm{MP}\!=\!Q, \mathrm{CP}\!=\!R) \\
&\quad - V^+(\mathrm{MP}\!=\!R, \mathrm{CP}\!=\!Q)
\end{aligned}
$$

The difference score

MP: advertises marginals $Q(x_i|x^*_{1:i-1})$

CP: chooses $x_i^* = \arg \left\{ \begin{smallmatrix} \min \\ \max \end{smallmatrix} \right\}_{x_i} \frac{Q(x_i|x^*_{1:i-1})}{R(x_i|x^*_{1:i-1})}$





$$\begin{aligned} S^+(Q,R) &= V^+(\mathrm{MP}=Q, \mathrm{CP}=R) \\ &\quad - V^+(\mathrm{MP}=R, \mathrm{CP}=Q) \end{aligned}$$

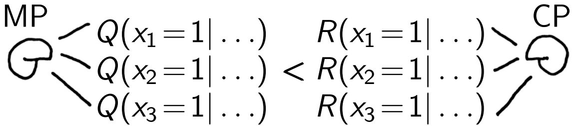
Note that

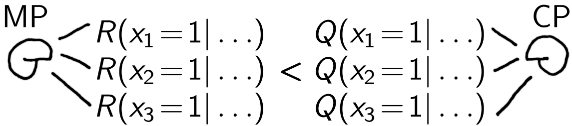
$$S^+(P,Q) \leq 0$$

The difference score

MP: advertises marginals $Q(x_i|x^*_{1:i-1})$

CP: chooses $x_i^* = \arg\left\{\begin{matrix}\min\\\max\end{matrix}\right\}_{x_i} \frac{Q(x_i|x^*_{1:i-1})}{R(x_i|x^*_{1:i-1})}$





$$\begin{aligned} S^-(Q,R) &= V^-(\mathrm{MP}\!=\!Q, \mathrm{CP}\!=\!R) \\ &\quad - V^-(\mathrm{MP}\!=\!R, \mathrm{CP}\!=\!Q) \end{aligned}$$

Note that

$$S^-(P,Q) \geq 0$$

The difference score

MP: advertises marginals $Q(x_i | x^*_{1:i-1})$

CP: chooses $x^*_i = \arg \left\{ \begin{matrix} \min \\ \max \end{matrix} \right\}_{x_i} \frac{Q(x_i | x^*_{1:i-1})}{R(x_i | x^*_{1:i-1})}$

MP

$$\begin{matrix} Q(x_1 = 1 | \ldots) & & R(x_1 = 1 | \ldots) \\ Q(x_2 = 1 | \ldots) & < & R(x_2 = 1 | \ldots) \\ Q(x_3 = 1 | \ldots) & & R(x_3 = 1 | \ldots) \end{matrix}$$

CP

MP

$$\begin{matrix} R(x_1 = 1 | \ldots) & & Q(x_1 = 1 | \ldots) \\ R(x_2 = 1 | \ldots) & < & Q(x_2 = 1 | \ldots) \\ R(x_3 = 1 | \ldots) & & Q(x_3 = 1 | \ldots) \end{matrix}$$

CP

$$\begin{aligned} S^-(Q, R) = \; & V^-(\text{MP} = Q, \text{CP} = R) \\ - \; & V^-(\text{MP} = R, \text{CP} = Q) \end{aligned}$$

Note that

$$S^-(P, Q) \geq 0$$

S^+ penalises overestimates, S^- penalises underestimates

The four-way score

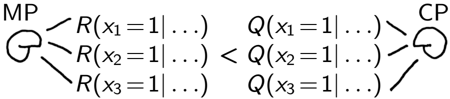
MP: advertises marginals $Q(x_i | x^*_{1:i-1})$

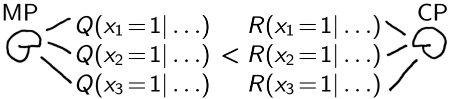
CP: chooses $x^*_i = \arg\left\{\begin{smallmatrix}\min\\ \max\end{smallmatrix}\right\}_{x_i} \frac{Q(x_i | x^*_{1:i-1})}{R(x_i | x^*_{1:i-1})}$

The four-way score

MP: advertises marginals $Q(x_i|x^*_{1:i-1})$

CP: chooses $x^*_i = \arg\left\{\begin{matrix}\min\\\max\end{matrix}\right\}_{x_i} \frac{Q(x_i|x^*_{1:i-1})}{R(x_i|x^*_{1:i-1})}$

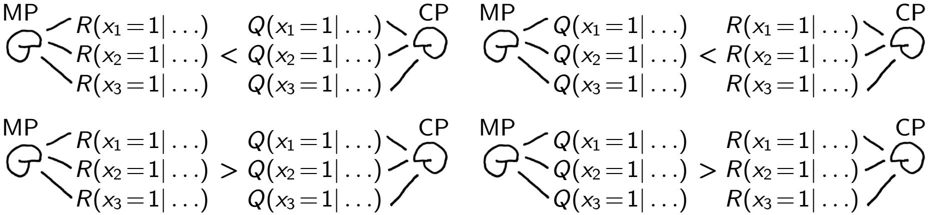




The four-way score

MP: advertises marginals $Q(x_i|x^*_{1:i-1})$

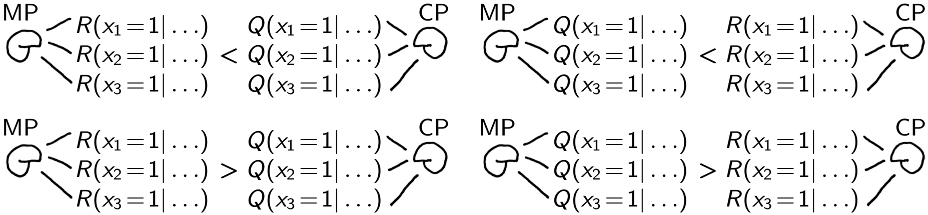
CP: chooses $x^*_i = \arg\left\{\begin{smallmatrix}\min\\ \max\end{smallmatrix}\right\}_{x_i} \frac{Q(x_i|x^*_{1:i-1})}{R(x_i|x^*_{1:i-1})}$



The four-way score

MP: advertises marginals $Q(x_i|x^*_{1:i-1})$

CP: chooses $x_i^* = \arg\left\{\begin{smallmatrix}\min\\ \max\end{smallmatrix}\right\}_{x_i} \frac{Q(x_i|x^*_{1:i-1})}{R(x_i|x^*_{1:i-1})}$

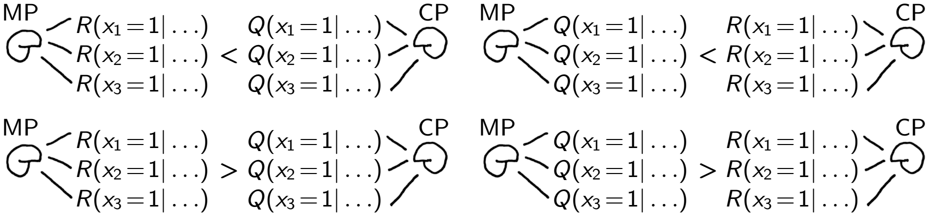


$$S_4(Q,R) = S^-(Q,R) - S^+(Q,R)$$

The four-way score

MP: advertises marginals $Q(x_i|x^*_{1:i-1})$

CP: chooses $x^*_i = \arg\left\{ {\min \atop \max} \right\}_{x_i} \frac{Q(x_i|x^*_{1:i-1})}{R(x_i|x^*_{1:i-1})}$



$$S_4(Q, R) = S^-(Q, R) - S^+(Q, R)$$

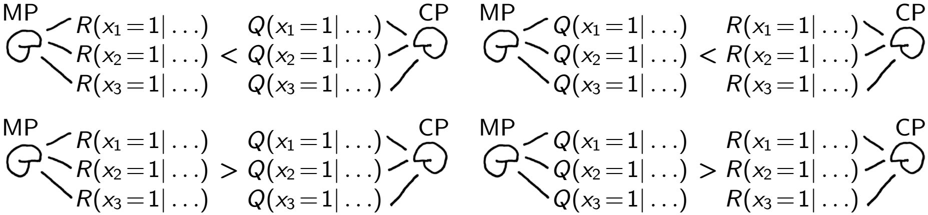
Note that

$$S_4(P, Q) \geq 0$$

The four-way score

MP: advertises marginals $Q(x_i | x^*_{1:i-1})$

CP: chooses $x^*_i = \arg \left\{ {\min \atop \max} \right\}_{x_i} \frac{Q(x_i | x^*_{1:i-1})}{R(x_i | x^*_{1:i-1})}$

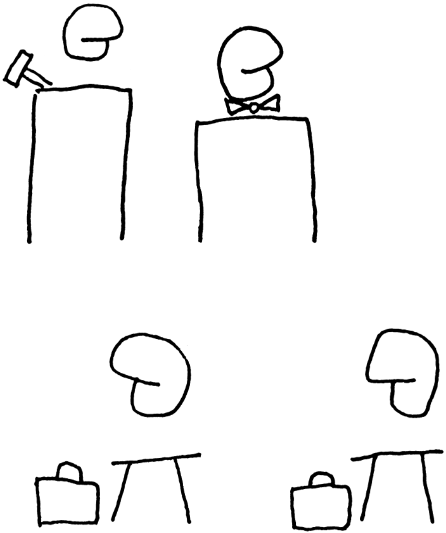


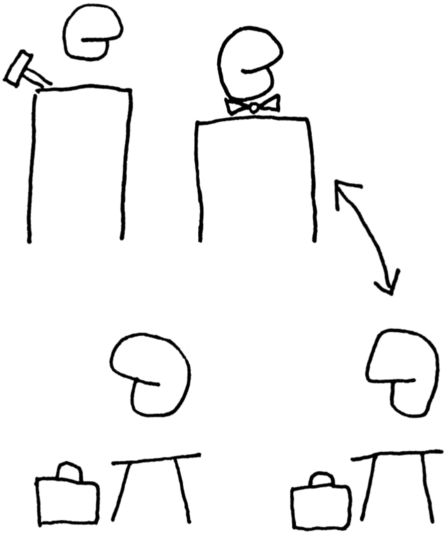
$$S_4(Q, R) = S^-(Q, R) - S^+(Q, R)$$

Note that

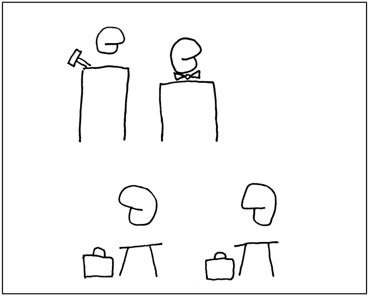
$$S_4(P, Q) \geq 0$$

... Too complicated?





Legal analogy



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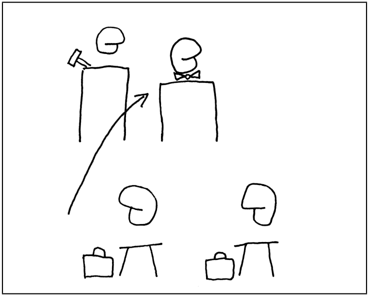
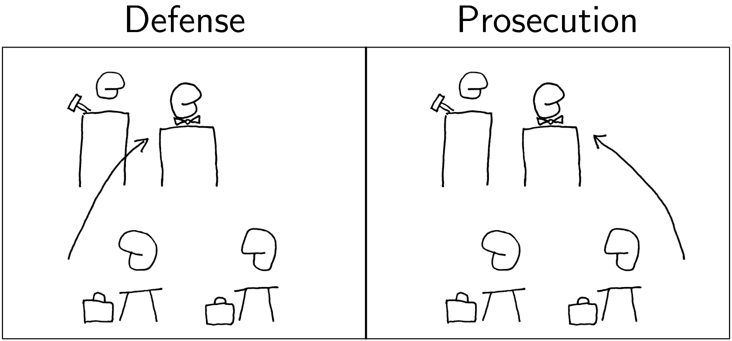
Defense



Legal analogy



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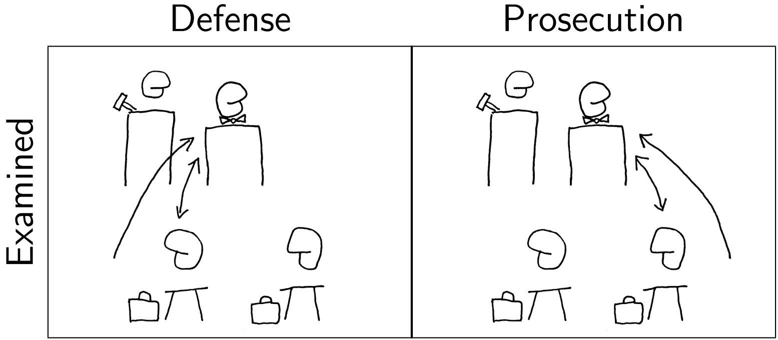

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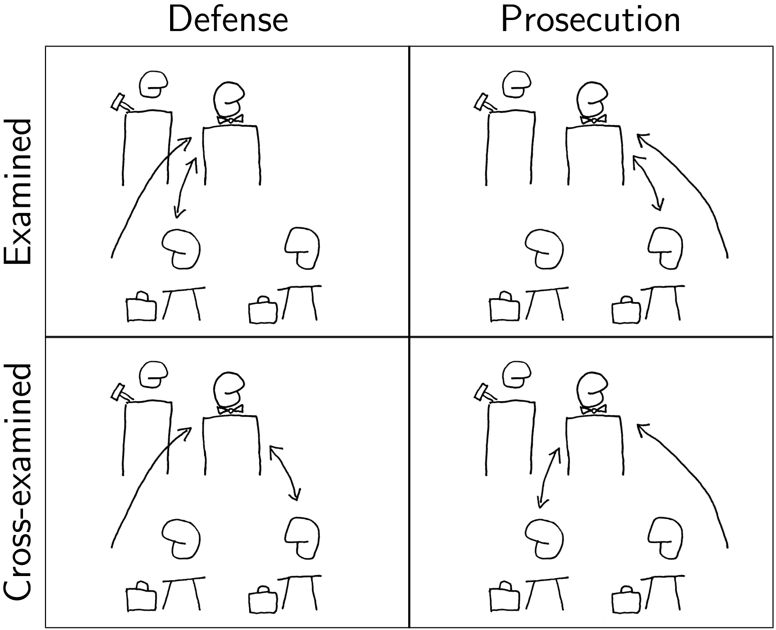

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What if neither Q nor R is exact?

Outline

Theoretical bounds for comparing approximations

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avg L_1 error: $\frac{1}{n}\sum_i \sum_{x_i} |Q(x_i) - P(x_i)|$

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Outline

Experimental results

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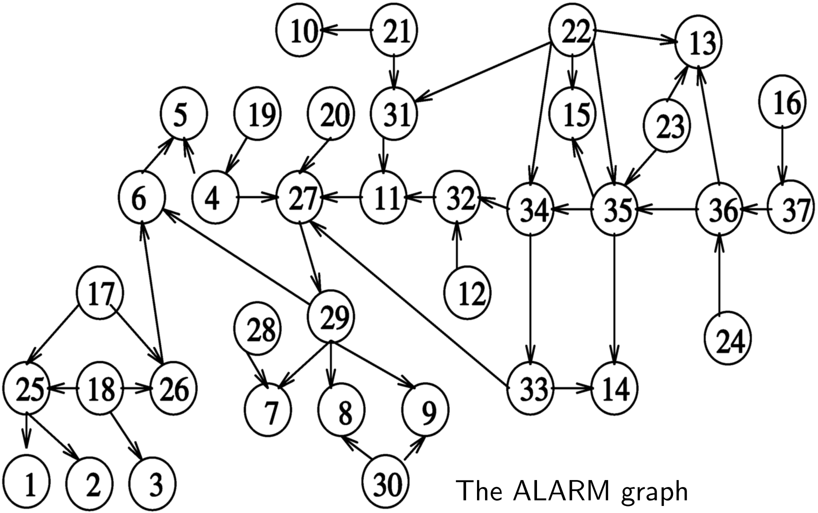

The ALARM graph
(Beinlich, 1989)
(diagram from Singh et al 1994)

Comparison of five approximations

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- Errors:

	avg L_1
LCBP	0.0001
TreeEP	0.0087
CBP	0.0111
BP	0.0163
Gibbs	0.0225

Comparison of five approximations

- Errors:

	avg L_1	avg $L_1^{\log}$
LCBP	0.0001	0.001
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Comparison of five approximations

- Errors:

	avg L_1	avg $L_1^{\log}$	max $L_1^{\log}$
LCBP	0.0001	0.001	0.017
TreeEP	0.0087	0.044	0.548
CBP	0.0111	0.054	1.256
BP	0.0163	0.071	1.642
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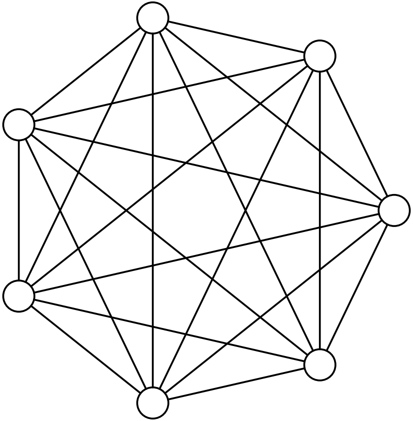
Gibbs	0.0225	0.211	0.830

- Pairwise S_4 scores:

	LCBP	TreeEP	CBP	BP	Gibbs
LCBP	0	5.3	13.8	22.8	13.0
TreeEP		0	8.4	13.5	4.0
CBP			0	27.6	3.7
BP				0	-4.0
Gibbs					0

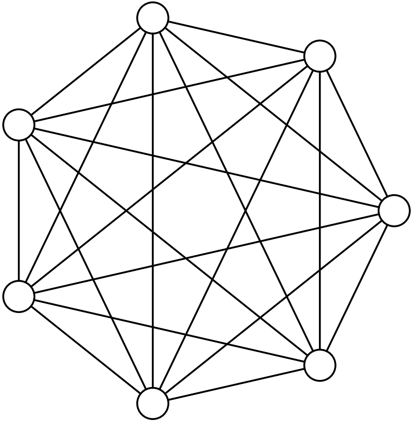
Random approximations and models

Random approximations and models



- Fully connected binary pairwise, 7 variables

Random approximations and models



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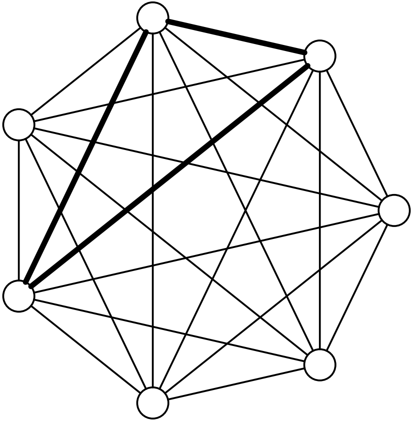

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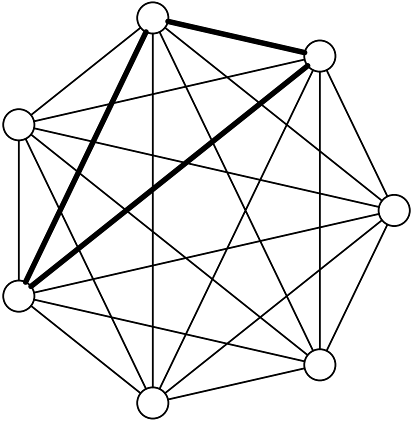

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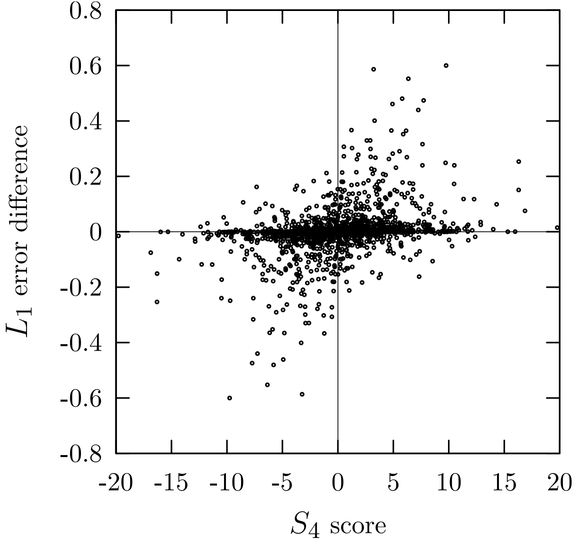
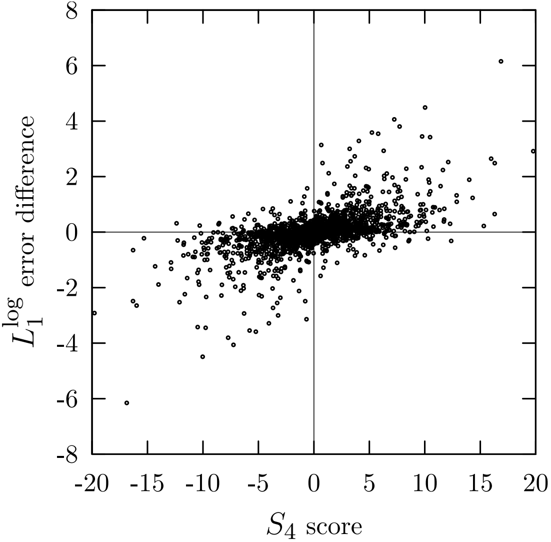
- (really HAK)

S_4 versus error



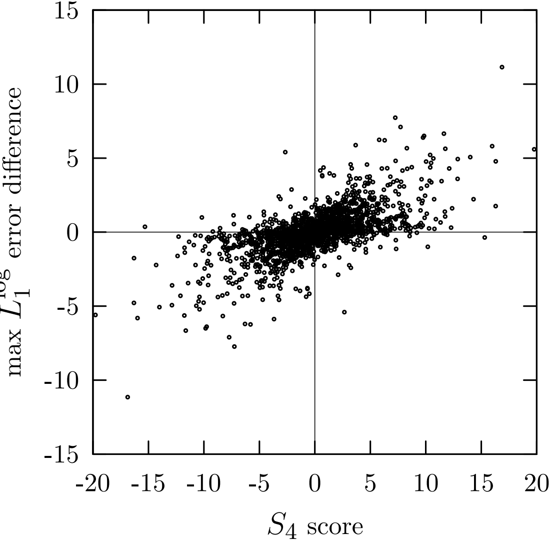
Average L_1 error: 64% agreement

S_4 versus error

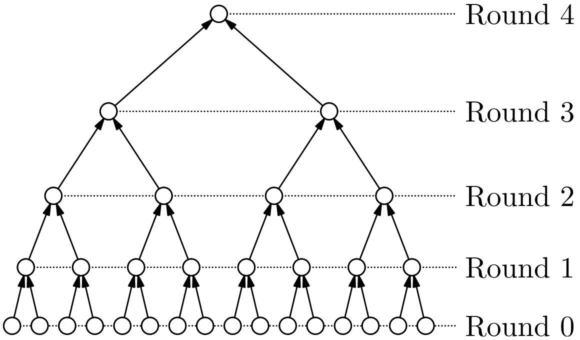


Average $L_1^{\log}$ error: 75% agreement

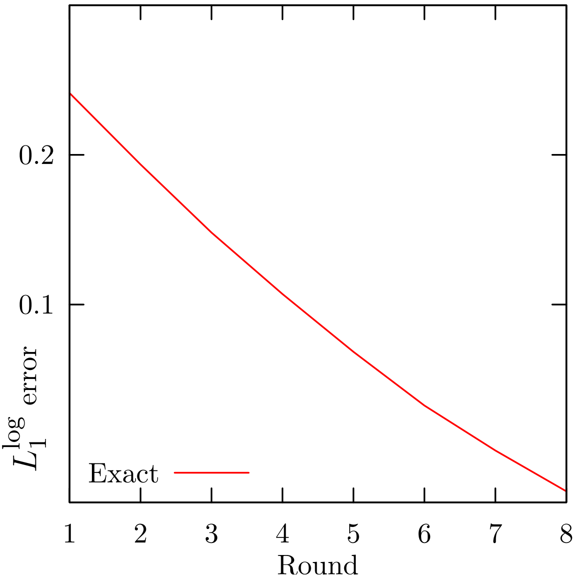
S_4 versus error



Maximum $L_1^{\log}$ error: 77% agreement


Round 4
Round 3
Round 2
Round 1
Round 0

Single-elimination tournament



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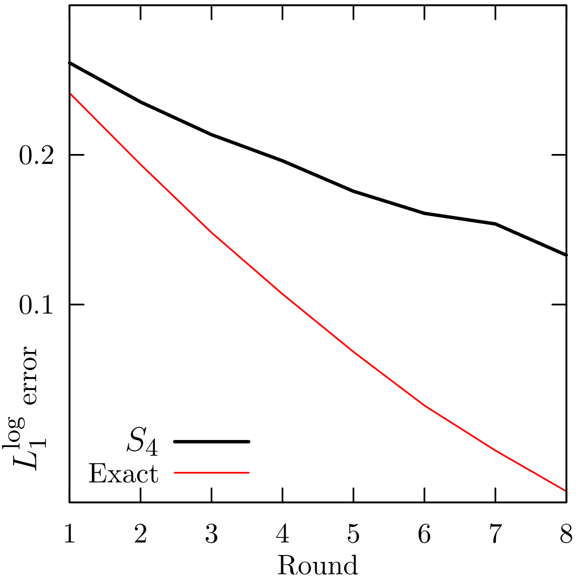

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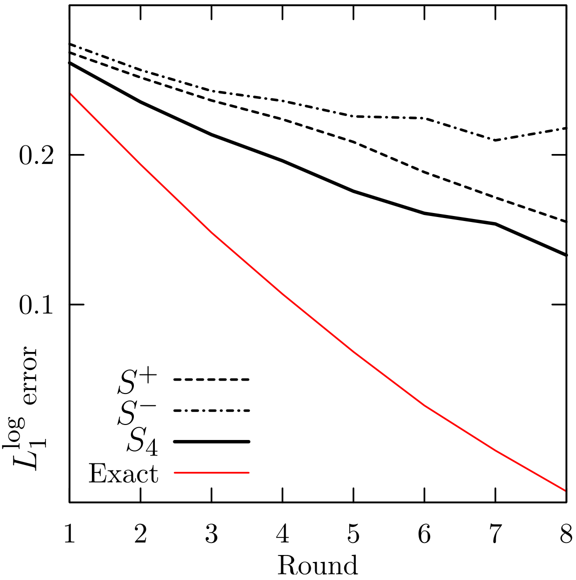

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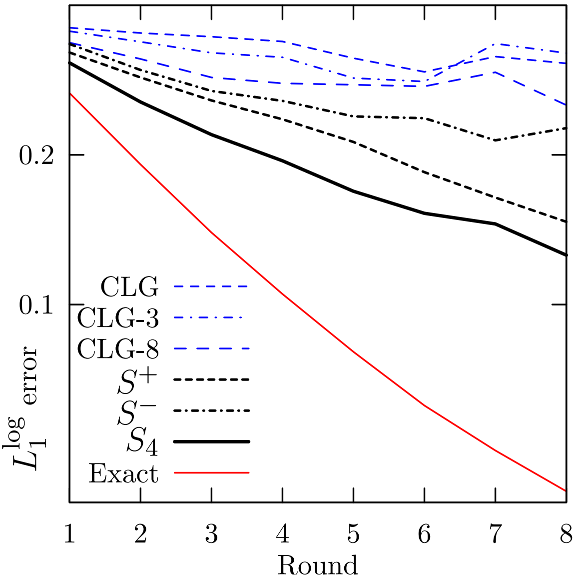

(Topsøe 1979)

Outline

Conclusion

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- Approximations with compilation (e.g. (Lowd 2010))